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Band structure and dynamic behaviors of Bose–Einstein condensates in Fourier-Synthesized optical lattices

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ABSTRACT

By employing a nonlinear three-mode model, we study the band structure of Bose–Einstein condensates in Fourier-Synthesized optical lattices, where the nonlinearity comes from the mean field treatment of interaction between atoms. In linear case, we present the band structure of the system. It is demonstrated that the energy band structure is strongly dependent on the value of relative phase of the two lattice harmonics. In the nonlinear case, we show that the eigenenergies as the functions of the quasi-momentum have a novel bowl structure in the middle energy level. It is found that there exist four critical values of interaction strength at which the band structure will undergo interesting changes. Furthermore, the stability of the eigenstate is also investigated.

1. Introduction

In recent years, Bose–Einstein condensates (BECs) in optical lattices have attracted enormous attention both experimentally and theoretically [1,2]. This is mainly because the lattice parameters and interaction strength can be manipulated by using modern experimental techniques. Researchers have discovered many novel phenomena, such as, nonlinear Landau–Zener tunneling, energetic and dynamical instability and the strongly inhibited transport of one-dimensional BEC in optical lattices [3–12]. One of the surprising discoveries is that the interaction between particles can influence the band structure dramatically [3,7].

Recently, ample interests have arisen in the field of transport properties of atoms and BECs subject to a Fourier-synthesized optical lattice (FS) [13,14]. The FS optical lattice is realized by superimposing a conventional standing wave potential of $\lambda/2$ spatial periodicity with a fourth-order multiphoton potential of $\lambda/4$ periodicity. The symmetric properties of such lattices can be controlled by the relative phase between the two standing waves, and the transport behaviors can be manipulated easily by the relative phase between the two spatial lattice harmonics. The transport properties of quantum objects subject to a periodic potential are determined by the band structure. So, the study of band structures for such FS optical lattices attracts many physicists. Here, we will focus on studying the band structure of BECs in a FS optical lattice.

In this paper, a nonlinear three-mode model is established based on the Gross–Pitaevskii (GP) equation with a FS optical lattice, where the nonlinearity comes from the mean field treatment of the interaction between condensate atoms. With this model, we reproduce the band structures of the lowest three levels for the linear case obtained in Ref. [13]. But in the nonlinear case, a more complicated band structure emerges with increasing interaction strength and a bowl structure is observed when the interaction exceeds a critical value. At the same time, several criteria are found in which the band structure will undergo interesting changes. In addition, the stabilities of the corresponding eigenstates are also studied.

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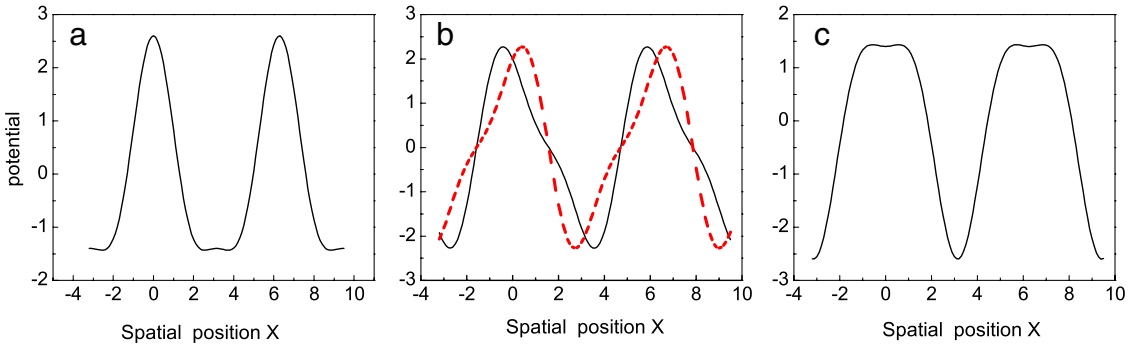

Fig. 1. Spatial potential for a periodic atom potential for different values of the phase α between lattice harmonics: (a) $\alpha = 0$, (b) $\alpha = \pi/2$ (solid line) and $\alpha = -\pi/2$ (dashed line), (c) $\alpha = \pi$. We have set $V_1 = 2$, $V_2 = 0.6$.

This paper is organized as follows. In Section 2, we obtain a dynamical equation describing the energy band properties of BEC in a FS optical lattice with three-mode approximation. In Section 3, the band structure of the periodic potential was derived by solving the eigenvalue equation of the system. There exists very particularly the energy band structure at the first Brillouin zone. Here, the main results and discussions are presented. In Section 4, we investigate the stability of the condensate by the Jacobian of the classical Hamiltonian. The summary and conclusion of our work are presented in Section 5.

2. Governing equations and three-mode approximations

We focus our attention on the situation that BEC is loaded into a one dimensional Fourier-synthesized optical lattices where the motion in perpendicular directions is confined. Here, the FS optical lattices are given by

$$V(x) = V_1 \cos(2k_0 x) + V_2 \cos(4k_0 x + \alpha) \tag{1}$$

where V_1 and V_2 denote the potential depths of two lattice harmonics respectively, k_0 is the wave number of the laser light which is used to generate the optical lattice, and α is the relative phase of two lattices. The shape and symmetry of the lattice depend on the relative phase of the two lattices, which is shown in Fig. 1. It is obvious that the lattice potential resembles a periodic sequence of hills for $\alpha = 0$ [Fig. 1(a)], the situation of spatial lattice potentials with the sawtooth-like structures are shown in Fig. 1(b) for $\alpha = \pm\pi/2$, and an array of potential dimples in the spatial lattice structure for $\alpha = \pi$ [Fig. 1(c)]. The values of the relative phase impact greatly on the band structure of the system which can be seen from the latter calculation. In the mean-field approximation, the dynamics of BEC can be modeled by the 1D-GP (Gross–Pitaevskii) equation in the comoving frame of the lattice,

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V_1\cos(2k_0x)\psi + V_2\cos(4k_0x+\alpha)\psi + \frac{4\pi\,\hbar^2\,a_s}{m}|\psi|^2\psi \tag{2}$$

where ψ is the wave function of the condensate, m is the mass of atoms, a_s is the two-body s-wave scattering length, k_0 is the wave number of the laser light which is used to generate the optical lattice. The first Brillouin zone is $[-k_0, k_0]$. For convenience, we cast Eq. (2) into the dimensionless form

$$\mathrm{i}\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi + v_1\cos(x)\psi + v_2\cos(2x+\alpha)\psi + c|\psi|^2\psi. \tag{3}$$

The dimensionless variables are scaled as,

$$x \sim 2k_0x, \qquad \psi \sim \frac{\psi}{\sqrt{n_0}}, \qquad t \sim \frac{4\hbar}{m}k_0^2t,$$

where the variable are scaled as

$$v_i = \frac{mV_i}{4\,\hbar^2\,k_0^2}, \qquad c = \frac{\pi n_0 a_s}{k_0^2}.$$

Here, n_0 is the average density of the BEC, v_i represents the strength of the potential and c denotes the atomic interaction. In our following discussions, we focus on the case of repulsive interaction between atoms. i.e. $c > 0$.

We consider following three-state model to describe the band structure of the system

$$\psi = a_l(t)\mathrm{e}^{\mathrm{i}(k-1)x} + a_0(t)\mathrm{e}^{\mathrm{i}kx} + a_r(t)\mathrm{e}^{\mathrm{i}(k+1)x}$$

where the total probability $|a_l|^2 + |a_0|^2 + |a_r|^2 = 1$, $\hbar k$ is the quasi-momentum. As shown in Refs. [14,15], the three-mode approximation can give an exact solution to the GP equation at the first Brillouin and qualitatively reproduces the behavior