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The two- and three-dimensional spin-3/2 random Blume–Capel model by the position space renormalization group

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ABSTRACT

We use the Migdal–Kadanoff renormalization group technique to study the spin-3/2 Blume–Capel model under a random crystal field, in the two- and three-dimensional cases. Studying the fixed points and the phase diagrams established, we find interesting results allowing us to understand the critical behavior of the system. In the two-dimensional case, the randomness, even in small amounts, removes completely the first order transition between the two ferromagnetic phases present, replacing it by a smooth continuation. Only the second order phase transitions occur. In the three-dimensional case, the first order phase transition disappears only at a certain threshold of randomness. Below this threshold, we observe the presence of an end-point where finishes the first order transition line inside the ferromagnetic phases. This end-point reaches $T = 0$ K at a critical value of probability, beyond which only the second order transitions occur.

1. Introduction

Many interesting studies have been devoted to the Blume–Capel model [1,2], initially introduced for the study of first order magnetic phase transitions. It is a spin-1 Ising model with a single anisotropic ion. Later, the Blume–Emery–Griffiths model [3], applied to the isotopic mixtures of helium ^{3}He–^{4}He, was proposed as a generalization of the Blume–Capel model. In these mixtures, the 3He and 4He atoms are respectively represented by the state $S = 0$ and $S = \pm 1$. The spin-1 Blume–Capel and Blume–Emery–Griffiths systems have been investigated by a variety of approximation methods, such as the variational methods [4,5], the effective field theory [6], the renormalization group techniques [7,8], the mean field approximation [9] and Monte Carlo simulations [10,11].

It is possible to extend the spin-1 Blume–Capel and Blume–Emery–Griffiths models by including higher spin values. The simplest extensions are probably the spin-3/2 Blume–Capel and Blume–Emery–Griffiths models, proposed to explain the tricritical properties in ternary fluids mixtures [12] and the magnetic and crystallographic phase transitions in some rare-earth compounds such as $DyVO_4$ [13]. Here also, several methods have been used, such as the mean field approximation [14], the effective field theory [15], and the techniques of renormalization group [16,17].

Many studies have been devoted for decades to systems subject to random fields [14,18,19] and in the view to understand their effect on the phase transitions. The presence of randomness produces remarkable impacts on the critical behavior of the systems. In two-dimensional systems, the first order phase transitions are removed totally for any amount of randomness. In three-dimensional systems, it is generally expected that the first order remains for weak disorder, disappearing completely

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at a critical value of randomness, above which only the second order phase transitions occur. It has been noted that only the presence of randomness is important; the exact form of the probability distribution does not play a fundamental role.

In our present study, we are interested in the spin-3/2 Blume–Capel model, on which we apply a random crystal field obeying to a probability distribution of two peaks. We use a position space renormalization group technique, namely the Migdal–Kadanoff [20,21] one. In its pure version, the spin-3/2 Blume–Capel model presents a second order transition line which separates the ferromagnetic and paramagnetic phases in both two- and three-dimensional cases; the two ferromagnetic phases are separated by a first order transition line which finishes at the second order transition line. We find that the introduction of randomness affects considerably the critical behavior of the system. Indeed, in the two-dimensional case, the randomness removes totally the first order phase transition, leaving only those of second order. But in the three-dimensional case, the first order transition is present at small amount of randomness, disappearing completely only beyond a certain threshold of randomness. Below this threshold, we observe the presence of an end-point where finishes the first order transition inside the ferromagnetic phases. This end-point reach $T = 0$ K at a critical value of the probability, beyond which only the second order transition occurs. Thus, we note the existence of a dimensional crossover indicating a qualitative difference in the critical behavior of the two- and three-dimensional random spin-3/2 Blume–Capel models.

We organize our article as follows. In Section 2, we treat the formalism of our method and we establish the Migdal–Kadanoff recursion equations. In Section 3, we present our results and discuss important points. Finally, we give a conclusion in the last section.

2. Model and technique

In order to study the spin-3/2 Blume–Capel model, we consider the following Hamiltonian:

$$-\beta H = J \sum_{i,j} S_i S_j + \sum_i \Delta_i S_i^2 \tag{1}$$

where the spins S_i, located at the site i on a discrete d-dimensional lattice, can take the four values $\pm 3/2$ and $\pm 1/2$. J is the reduced bilinear interaction and Δ_i is the crystal field at the site i. The first summation is over all nearest neighbor pairs of the lattice and the second one over all sites.

In the Blume–Capel model, the reduced biquadratic interaction K is equal to zero, but we will take it into account due to the renormalization group technique we are using. We also introduce two additional interactions, C and F, to obtain self-consistent recursion relations. Thus, the Hamiltonian we will effectively use in the remainder of our work is as follows:

$$-\beta H = J \sum_{i,j} S_i S_j + \sum_i \Delta_i S_i + C \sum_{i,j} \left(S_i S_j^3 + S_i^3 S_j \right) + F \sum_{i,j} S_i^3 S_j^3. \tag{2}$$

We are not concerned here about the physical meaning of the interactions C and F, they are added only for a purely technical purpose in order to preserve the parameters space renormalization. Indeed, the renormalization does not keep in general the parameters space of the Hamiltonian, what constitutes an anomaly being able to cause physical aberrations. For example, in the Blume–Emery–Griffiths model, the three parameters J, K and Δ are insufficient to stabilize the ferrimagnetic phase which has been obtained by all the methods of effective fields.

The different phases of the Blume–Capel model can be characterized by two order parameters, the magnetization $m = \langle S_i \rangle$ and the quadrupolar momentum $q = \langle S_i^2 \rangle$. When $m = 0$, we have two paramagnetic phases, referred to as $P_{3/2}$ and $P_{1/2}$, characterized respectively by $q > 5/4$ and $q < 5/4$. When $m \neq 0$, we have two ferromagnetic phases labeled $F_{3/2}$ and $F_{1/2}$, distinguished respectively by $q > 5/4$ and $q < 5/4$.

The crystal field Δ_i is subject to randomness and obeys to a probability distribution $P(\Delta_i)$ with two peaks that is given by

$$P(\Delta_i) = p\delta(\Delta_i + \Delta) + (1 - p)\,\delta(\Delta_i - \Delta). \tag{3}$$

To have a more reliable qualitative appreciation of the phase transitions characteristics, we use an approximation of the real space renormalization group, namely the Migdal–Kadanoff one, which combines decimation and bond shifting and is tractable in all space dimensionalities. In order to implement the renormalization machinery, we consider a one-dimensional chain of four spins S_1, S_2, S_3 and S_4, coupled by the interactions J, K, C and F; Δ_1, Δ_2, Δ_3 and Δ_4 are the crystal fields at each site of the chain. The spatial factor rescaling, denoted by b, is chosen as an odd integer to keep the possible sublattice symmetry breaking character of the system. In our present study, we take $b = 3$. The crystal field is a local interaction and must be adapted with the renormalization procedure by transforming it into a bond, what is possible by performing an equal sharing of Δ_i along the $2d$ bonds leading to the site i. Furthermore, we have to take into account the coordination number of the site i in the crystal field term. With these considerations, we can write the reduced Hamiltonian of the four spins cluster as

$$\begin{aligned} -\beta H = {} & J(S_1S_2 + S_2S_3 + S_3S_4) + K\left(S_1^2S_2^2 + S_2^2S_3^2 + S_3^2S_4^2\right) + \frac{\Delta_1 S_1^2 + \Delta_4 S_4^2}{2d} + \frac{\Delta_2 S_2^2 + \Delta_3 S_3^2}{d} \\ & + C\left(S_1S_2^3 + S_1^3S_2 + S_2S_3^3 + S_2^3S_3 + S_3S_4^3 + S_3^3S_4\right) + F\left(S_1^3S_2^3 + S_2^3S_3^3 + S_3^3S_4^3\right). \end{aligned} \tag{4}$$