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Stability analysis of a multi-phase car-following model

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ABSTRACT

This paper presents a numerical stability analysis of a multi-phase car-following model under mild to severe disturbances. The results show that local stability was always conformed. An asymptotically unstable region was found for traffic in congested states. One of the previously calibrated boundary conditions for close-following situations was found to be in conflict with the stable condition required by the car-following model, which had attributed to speed oscillations during transition of the traffic from a non-congested to a congested state. Suggestions were made to the choice of model parameter values to meet the stability conditions and ways to improve the model.

1. Introduction

Traffic flow has attracted multidisciplinary interests in recent years due to the increasing traffic congestion problem on highways and the complexity of the traffic flow system [1–7]. Traffic behaviour has been studied by microscopic and macroscopic models, and by a variety of approaches ranging from car-following models [2,3], cellular automation models [4,5], to gas kinetic and hydrodynamic models [6,7].

Car-following model is a microscopic description of the behaviour of vehicles following one another in a single stream of traffic [1–3], and is one of the fundamental building blocks of microscopic representation of traffic flow. There are many formulations of car-following models and some attempt to generalise them [1]. Some of the earliest car-following models were developed by General Motors (GM) based on vehicle-following data collected on their test-tracks [2]. The GM models represent the response of a following vehicle in terms of its acceleration and deceleration to the stimulus it received from the vehicle ahead and its driver's sensitivity. The stimulus is usually represented as a function of the relative velocity and spacing between the two vehicles (e.g. Refs. [2,3]). These types of models are often referred to as *psycho-physical car-following models* in reference to their combined representation of drivers' reactions with the laws of physics on the dynamic equation of vehicle motion.

Another type of models, so called *safety-distance models*, are based on the simple idea that whatever the following vehicles do, they want to keep a safe distance behind so that they do not collide with the vehicle(s) in front. One of the most well-known safety-distance based car-following models is the Gipps model [8], which is widely adopted in traffic micro-simulation software [9,10].

Most of these models represent a single state of traffic, i.e. there is a single, fixed rule for the car-following behaviour throughout. Moreover, many of the models including the GM models, are based on empirical investigations carried out at relatively low speeds (mostly in the region of 30–60 km/h), which may not reflect more general car-following behaviour of traffic on high-speed road networks.

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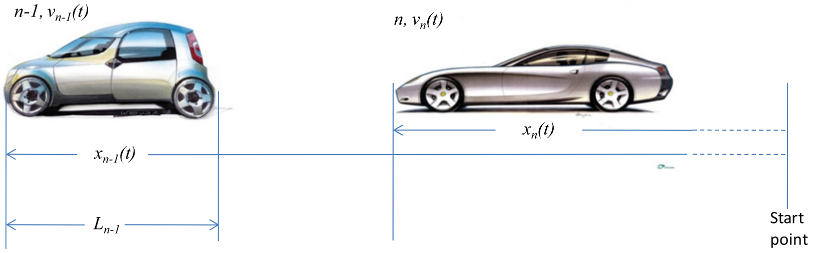

Fig. 1. Definitions used in a basic car following situation.

In recent years, the technical progress and the instrumentation of many of our highway networks have enabled collection and analysis of large sets of empirical traffic flow data and across a wide range of different traffic conditions [11,12]. Studies of such highway traffic flows have revealed the existence of different traffic phases ranging from free-flow, synchronised traffic with lower velocity but still high flows, to complete flow breakdown and traffic jams [1,13]. This has led to the development of *multi-regime car-following models* which apply different car-following rules to represent drivers adapting to different driving behaviour under different traffic conditions (e.g. Refs. [14,15]).

Two phenomena of highway traffic have received particular attention recently; namely *close-following* behaviour and *traffic hysteresis*. Considerably closer following than often found in urban traffic is observed when highway traffic is near capacity, but before the breakdown [16,17]. This is characterised by vehicles driving at very high speed but keeping very small gaps, sometimes with a time gap as low as 0.8 s [18]. It is believed that close-following is one of the main causes of traffic instability and therefore traffic jams [11,19,20]. Traffic hysteresis is a phenomenon characterised by a loop structure from empirical observed flow-occupancy plots, where the capacity of a traffic flow recovering from a flow breakdown does not reach the capacity before the breakdown [21–24].

Wang et al. [25] proposed a multi-phase car-following model which explicitly included a close-following phase and introduced the concept that drivers' reaction times are different during different phases. The model was shown to be capable of reproducing the full spectrum of traffic states, and in particular close-following and traffic hysteresis. The key model parameters were calibrated using aggregated traffic detector data [26]. The main contribution of the current paper is to study the stability properties of this model using a numerical simulation method.

The paper is organised as follows: Section 2 of the paper describes briefly the model concerned. Section 3 introduces the concept of local and asymptotic stabilities and presents results showing the performance of the model in the stability tests. An unstable, oscillatory behaviour of the model was found and questions were raised on the model parameter values used. Section 4 discusses the implications and suggests ways the model can be improved. Finally conclusions are drawn in Section 5.

2. A multi-phase car-following model

The model by Wang, Liu and Montgomery [25], hereafter the WLM model, was designed to represent traffic flow on high-speed freeway networks, in particular to capture some of the key characteristics of freeway traffic flow such as the close-following behaviour and traffic hysteresis phenomenon.

For the description and the formulation of the WLM model, we consider a simple car-following situation as illustrated in Fig. 1 where vehicle n follows vehicle $n-1$ in a single stream of traffic. The variable $x_n(t)$ denotes the position vehicle n as measured from an arbitrary starting point, whilst $v_n(t)$ is its velocity at time t. L_{n-1} is the "effective" size of vehicle $n-1$, which includes the physical length of the vehicle plus a safe margin.

The WLM model was built on the concept that drivers in different traffic conditions (or states) behave differently. This concept was represented in the model by drivers applying different accelerations and reaction times in different states. The states considered were: traffic build-up from free-flow towards congestion, close-following, flow breakdown and recovery.

The model assumed that, as the traffic was getting congested, drivers' behaviour would change and they would become more alert to their surroundings. This change was characterised in the model by a critical driving speed, v_c at 50 km/h [17]. Above this speed threshold (i.e. when traffic moves more freely), drivers were considered to be in a *non-alert state* with longer reaction times and lower acceleration and deceleration. Below this critical speed (when traffic becomes congested), drivers were considered to be in an *alert state* with shorter reaction times and higher acceleration and braking power. After a highly alert state during congestion and flow breakdown and during the recovery state, the drivers were assumed to want to relax a little bit and return back to the non-alert state with longer reaction time and lower acceleration/deceleration.

Situated in between the non-alert and alert states during traffic build-up is a close-following state. The exact definition of the close-following state is described later.

In its mathematical formulation, the WLM model combines the idea of the safe-distance model of Gipps [8] for alert and non-alert states, with the model of Leutzbach and Wiedemann [14] for close-following state. The WLM model then covers the full spectrum of traffic states and allows vehicles to move from one state to another in a continuous space–time domain.