Physica A 392 (2013) 2680–2687

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Damage spreading in a driven lattice gas model

M. Leticia Rubio Puzzo [a,*], Gustavo P. Saracco [a], Ezequiel V. Albano [b,c]

[a] Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas (INIFTA), UNLP, CCT La Plata - CONICET, c.c. 16, Suc. 4, (1900) La Plata, Argentina
[b] Instituto de Física de Líquidos y Sistemas Biológicos (IFLYSIB), UNLP, CCT La Plata - CONICET, calle 59 nro 789, (1900) La Plata, Argentina
[c] Departamento de Física, Facultad de Ciencias Exactas, UNLP, La Plata, Argentina

ARTICLE INFO

Article history:
Received 3 October 2012
Received in revised form 23 November 2012
Available online 17 January 2013

Keywords:
Damage spreading
Driven lattice gas
Monte Carlo simulations

ABSTRACT

We studied damage spreading in a Driven Lattice Gas (DLG) model as a function of the temperature T, the magnitude of the external driving field E, and the lattice size. The DLG model undergoes an order–disorder second-order phase transition at the critical temperature $T_c(E)$, such that the ordered phase is characterized by high-density strips running along the direction of the applied field; while in the disordered phase one has a lattice-gas-like behavior. It is found that the damage always spreads for all the investigated temperatures and reaches a saturation value D_{sat} that depends only on T. D_{sat} increases for $T < T_c(E = \infty)$, decreases for $T > T_c(E = \infty)$ and is free of finite-size effects. This behavior can be explained as due to the existence of interfaces between the high-density strips and the lattice-gas-like phase whose roughness depends on T. Also, we investigated damage spreading for a range of finite fields as a function of T, finding a behavior similar to that of the case with $E = \infty$.

1. Introduction

The statistical mechanics of equilibrium phenomena is very useful for understanding the thermodynamic properties of many-particle systems from a microscopical point of view. From its beginnings up to now, new developments and theories have enriched it, culminating in the renormalization-group approach [1,2]. In nature, most many-particle systems are under far-from-equilibrium conditions, and yet there is not a well-established theoretical framework to treat them, as in the case of their equilibrium counterpart.

In order to overcome this shortcoming, many attempts have been made to gain some insight into the far-from-equilibrium behavior, e.g. by studying simple models that are capable of capturing the essential non-equilibrium behavior. Within this context, one of the best known paradigms of far-from-equilibrium systems is the two-dimensional driven lattice gas (DLG) model proposed by Katz, Lebowitz and Spohn [3]. This model consists of a set of particles located in a two-dimensional square lattice in contact with a thermal reservoir. Particles exchange places with nearest-neighbor empty sites according to spin exchange, i.e. the Kawasaki dynamics. Also, an external drive is applied, causing the system to exhibit non-equilibrium stationary states (NESS) in the limit of large evolution times. If a half-filled two-dimensional system is considered (as in this paper), and for low enough temperatures, the DLG model develops an ordered phase characterized by strips of high particle density running along the driving direction [4]. However, by increasing the temperature a second-order non-equilibrium phase transition into a disordered (gas-like) phase takes place. The critical temperature (T_c) depends on the value of the driving field E, and in the limit of $E \to \infty$ one has $T_c \simeq 1.41T_O$, where T_O is the Onsager critical temperature of the Ising model [5]. The critical behavior of the DLG model has been studied by using many different techniques [6], such

* Corresponding author. Tel.: +54 221 4257430.
E-mail address: lrubio@inifta.unlp.edu.ar (M.L. Rubio Puzzo).

0378-4371/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2013.01.018

as field theoretical calculations [7–10], Monte Carlo simulations [11–15], finite-size scaling methods [13,14], and short-time dynamic scaling [16–18], but the complete understanding of this model is still lacking and has originated a long-standing controversy [6–18]. It should be noticed that Monte Carlo studies of the DLG model are mostly focused on understanding the critical behavior of NESS for half-filled lattices.

From the theoretical point of view, it is interesting and challenging to study the dynamic evolution of a very small perturbation in a non-equilibrium system. One way to do this is to apply the concept of *damage spreading*. Originally introduced by Kauffman [19,20] to study biological systems, years later it was applied by Stanley et al. [21] and Derrida et al. [22] to study physical properties in non-deterministic systems such as Ising Model or Spin Glasses. This method is based on the point-to-point comparison between two slightly different configurations of a system that are allowed to evolve simultaneously. In order to achieve these configurations, one sample is initially perturbed by slightly changing its configuration, so that it is called the "damaged" sample, while the original sample remains unperturbed. Then the time evolution of the perturbation, defined as the difference between configurations, is followed. In the long-time limit, the perturbation can either survive or vanish, according to the values of the control parameters of the system.

Damage spreading studies were originally applied to the Ising model, spin glasses and cellular automata [19–22], but they have also been applied to magnetic systems such as the Potts model with q-states, Heisenberg and XY models, two-dimensional trivalent cellular structures, biological evolution, non-equilibrium models, opinion dynamics, ZGB model and small world networks (see e.g. Ref. [23] and references therein, and for more recent results see Refs. [18,24–30]).

Within the broad context discussed above, the goal of our work is to give an overall description of the damage spreading process in the DLG model as a function of the control parameters, i.e. the temperature and the field magnitude, and also of the lattice dimensions.

The manuscript is organized as follows: the DLG model is described in Section 2, while in Section 3 details of the damage spreading technique are explained. The results are presented and discussed in Section 4, and finally our conclusions are stated in Section 5.

2. The model

The DLG model [3] is defined on the square lattice of size $L \times M$ with periodic boundary conditions along both directions. The driving field, E, is applied along the M-direction. Each lattice site can be empty or occupied by a particle. If the coordinates of the site are (i, j), then the label (or occupation number) of that site can be $\eta_{ij} = \{0, 1\}$. The set of all occupation numbers specifies a particular configuration of the lattice. The particles interact among them through a nearest-neighbor attraction with positive coupling constant ($J > 0$). So, in the absence of any field, the Hamiltonian is given by

$$H = -4J \sum_{\langle ij;i'j' \rangle} \eta_{ij}\eta_{i'j'}, \tag{1}$$

where $\langle\cdot\rangle$ means that the summation is made over nearest-neighbor sites only.

The attempt of a particle to jump to an empty nearest-neighbor site, W_{jump}, is given by the Metropolis rate [31] modified by the presence of the driving field, that is,

$$W_{jump} = \min[1,\ e^{[\Delta H - \epsilon_1 E]/k_B T}], \tag{2}$$

where k_B is the Boltzmann constant, T is the temperature of the thermal bath, ΔH is the energy change after the particle–hole exchange, and $\epsilon_1 = (1,\ 0,\ -1)$ assumes these values when the direction of the jump of the particle is against, orthogonal or along the driving field E, respectively. The field is measured in units of J and temperatures are given in units of J/k_B. In this context, the critical temperature for the case with $E = \infty$ is $T_c \simeq 3.2$. The dynamics imposed does not allow elimination of particles, so the number of them is a conserved quantity. Also, in the absence of a driving field, the DLG model reduces to the Ising model with conserving (i.e. Kawasaki) dynamics. For further details of the DLG model, see e.g. Refs. [4,6,32].

3. The damage spreading method

The Damage Spreading (DS) method was initially introduced [19–22] to investigate the effects of tiny perturbations introduced in the initial condition of physical systems on their final stationary or equilibrium states. In order to implement the DS method in computational simulations [33,34], two configurations or samples S and S', of a given stochastic model, are allowed to evolve simultaneously. Initially, both samples differ only in the state of a small number of sites. Then, the difference between S and S' can be considered as a small initial perturbation or damage. In order to give a quantitative measure of the evolution of the perturbation, the "Hamming" distance or damage $D(t)$ is defined as

$$D(t) = \frac{1}{N} \sum_{i,j}^{N} 1 - \delta_{\eta_{ij}(t),\eta'_{ij}(t)}, \tag{3}$$

where $N = L \times M$ is the total number of sites in the lattices, $\eta'_{ij}(t)(\eta_{ij}(t))$ is the occupation number of site (i, j) in the sample $S'(S)$, and $\delta_{\eta_i(t),\eta'_i(t)}$ is the Kronecker delta function. The sum runs over all sites of both samples, so $0 \leq D(t) \leq 1$.