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Advantages of hopping on a zig-zag course

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Abstract

We investigate self-moving particles which prefer to hop with a certain turning angle equally distributed to the right or left. We assume this turning angle distribution to be given by a double Gaussian distribution. Based on the model of Active Brownian particles and we calculate the diffusion coefficient in dependence on the mean and the dispersion of the turning angles. It is shown that bounded distribution of food in patches will be optimally consumed by the objects if they hop preferably with a given angle and not straight forwardly.

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1. Introduction

In 1827, when the British botanist Robert Brown discovered the erratic motion of small particles immersed in a liquid, he considered them first as living entities. Later on Einstein and Smoluchowski [1] have shown that the behavior of Brownian particles are due to the molecular agitation, i.e., collisions from the side of the solvent molecules only.

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The most fascinating formulation was given by Langevin [2] who added random forces to the equation of motion, in addition to the viscous forces acting on the particles in the fluid. Recently, this latter formulation was used to study active particles which are self-moving particles thanks to some energy supply and, thus, remember the entities as they were considered by Brown. This concept of active Brownian particles is easy, negative friction at smaller velocities resembles a pump of energy. This energy is transformed into kinetic one up to some velocity (for higher ones, dissipation wins again) and the particles starts to be moving along their given direction. In case of additionally acting forces, in particular random ones, the individuals change the direction as well as the modulus of their velocity.

Several interesting phenomena were studied in the past. Simple models of active particles were studied already in many earlier works [3–6]. For broader reviews one might refer to Refs. [7–9]. Other previous versions of Active Brownian particle models [10,11] consider more specific activities, such as environmental changes and signal-response behavior. In these models, the Active Brownian particles (or active walkers, within a discrete approximation) are able to generate a self-consistent field, which in turn influences their further movement and physical or chemical behavior. This non-linear feedback between the particles and the field generated by themselves results in an interactive structure formation process on the macroscopic level. Hence, these models have been used to simulate a broad variety of pattern formations in complex systems, ranging from physical to biological and social systems [10,12,13].

The concept of Active Brownian particles was also applied to the motion of small water flees, so called *Daphnia* [9,14] where it can also explain some collective swarming and curling of the animals and in light shafts [15,16]. These *Daphnia* exhibit another property during their motion as was recently detected in measurements [15]. During a small time step they hop with some preferred angle to the left or right of their previous motion. Hence the direction of motion is due to some given distribution of turning angles which determines the preferred direction after the time step.

In Ref. [17] we applied a random walk model to find the effective diffusion coefficient of the random motion with preferred turning angles. For this purpose we have used an expression derived by Kareiva and Shigesada (see Ref. [18] for details) and applied it to the experimental observations [15]. Here we develop a continuous model which is based on the stochastic equations of motion in two dimensions. Again we find a decrease of diffusion with the mean turning angle and a non-monotonous dependence on the dispersion of angles.

In addition, we let the particle consume some non-moving food with constant rates. If this food is distributed in bounded regions, it would be suggestive to hop with a preferred turning angle. That's why the food consumption in a given time is maximized as will be shown in the last section.

2. Equations of motions of Active Brownian particles

The heart of Active Brownian particles is an energy pump modeled by negative dissipation at small velocities. Thus we suppose that the relaxation coefficient of the