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Asymmetric confinement in a doubly modulated bistable device

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Abstract

A Brownian particle hopping in a symmetric double-well potential may be statistically confined into a single well by the simultaneous action of two periodic input signals, one tilting the minima, the other one modulating the barrier height. Such a basic mechanism for asymmetric confinement can be conveniently maximized by tuning the input signal parameters (frequencies, phase lag, amplitudes), thus leading to a resonant localization of the particle in a target potential well.

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A Brownian particle bound by a bistable potential diffuses symmetrically between the potential minima; this is the case of Kramers' dynamics [1], where the particle is activated by thermal fluctuations, as well as of stochastic resonance (SR) [2,3], where the particle escape over the potential barrier is controlled by the interplay of thermal fluctuations and external periodic drive(s). In both cases, the time-averaged particle distribution density peaks symmetrically in correspondence with the potential minima; particle localization into one well is customarily achieved by applying an external static bias that breaches the symmetry of the system [4].

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In a number of applications experimenters are interested in confining the diffusing particle around one stable configuration and then manipulating it by means of various techniques; recent examples include magnetic flux microscopy [6], laser traps [5], quantum device design [7], to mention but a few. However, in most circumstances adding an external bias to the system at hand is inconvenient; hence, the need for an alternate approach to the confinement problem.

In the present paper confinement in a noisy bistable device is achieved without apparent symmetry breaking. A Brownian particle driven by a white, zero-mean Gaussian noise (mimicking thermal fluctuations) and, possibly, by a sinusoidal force with angular frequency Ω_1, may be localized into one state by modulating the potential barrier separating the two degenerate states. To this purpose one can input a sinusoidal (multiplicative) control signal with frequency Ω_2 and appropriate phase-lag. The corresponding steady distribution develops one prominent peak, whose relative magnitude hits a maximum for optimal values of the input parameters; in whose degenerate state the particle gets trapped in, depending on the switch-on phase of the modulating signal.

The key mechanism underlying the phenomenon of asymmetric confinement is well illustrated by the simplified case of an overdamped Brownian particle of coordinate $x(t)$ diffusing in a quartic double-well potential $V(x) = -ax^2/2 + bx^4/4$, with $a, b > 0$, subjected to a zero-mean Gaussian noise $\xi(t)$ and two low-amplitude rectangular signals $\varepsilon_i(t) = \varepsilon_i \,\mathrm{sgn}[\cos(\Omega_i t + \phi_i)]$, with $i = 1, 2$ and period $T_i = 2\pi/\Omega_i$, namely

$$\dot{x} = a[1 + \varepsilon_2(t)]x - bx^3 + ax_0\varepsilon_1(t) + \xi(t) \tag{1}$$

with $\langle \xi(t)\xi(0)\rangle = 2D\delta(t)$. Here, the additive signal $\varepsilon_1(t)$ tilts the potential sidewise, whereas the control signal $\varepsilon_2(t)$ sets the symmetric barrier separating the degenerate unperturbed minima $\pm x_0 = \pm\sqrt{a/b}$, to a high/low height $\Delta V_\pm = \Delta V_0(1 \pm \varepsilon_2)^2$, with $\Delta V_0 = a^2/4b$.

The steady (time-averaged) distribution density $P(x)$ of the stochastic process (1) has been computed by standard numerical simulation. In Fig. 1a our numerical results for $\Omega_1 = \Omega_2$ and $\phi_1 = \phi_2 = 0$ exhibit a marked asymmetry, corresponding to a stochastic localization of the particle to the left, i.e., in the negative potential well. The qualitative interpretation of such occurrence is straightforward: As long as the driving force $\varepsilon_1(t)$ points to the right, the barrier is set to its larger value ΔV_+, so that the Brownian particle takes a relatively long time to jump into the more stable well to the right (from where it can hardly escape); vice versa, as the additive force reverses sign, the barrier switches to its lower value ΔV_-, thus speeding up the right-to-left escape process. As a result, the spatial distribution density $P(x)$ tends to accumulate around $-x_0$. Of course, shifting the relative phase $\phi_2 - \phi_1$ by π is equivalent to changing $x \to -x$ and thus reversing asymmetry, as shown in Fig. 1a.

The *gating* mechanism [8] invoked here is expected to become increasingly efficient as D is lowered below ΔV_0. In Fig. 1b the subtracted asymmetry factor $\sigma \equiv P_-/P_+ - 1$, with $P_\pm \equiv \langle \int_0^\infty P(\pm x, t)\,\mathrm{d}x\rangle_t$ and $\langle \cdots \rangle_t$ denoting the stationary time average taken over one forcing cycle, diverges exponentially for $D \to 0$ and tends to zero (no confinement) according to a power law for $D \to \infty$.