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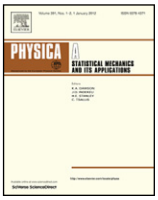

Role of recommendation in spatial public goods games

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ABSTRACT

We study the role of recommendation in a co-evolutionary public goods game in which groups can recommend their members for establishment of new relationships with individuals outside the current group according to group quality. Intriguingly, for square lattices and ER graphs there exists optimal group quality for recommendation that induces positive feedback between cooperation and recommendation. Snapshots of spatial patterns of cooperators, defectors, recommended cooperators and recommended defectors show that if group quality is appropriate for recommendation, cooperation and recommendation can simultaneously emerge. Moreover, we find that local recommendation improves cooperation more than global recommendation. As an extension, we also present results for Barabási–Albert networks. The positive effect of recommendation on cooperation for Barabási–Albert networks is independent of group quality. Our results provide an insight into the evolution of cooperation in real social systems.

1. Introduction

Cooperation among selfish individuals is ubiquitous in human society. This plays a key role in the evolution of humans and human society. The evolution of cooperation is the focus of increasing attention in various disciplines [1–4]. The prisoner's dilemma game, the snowdrift game and the ultimatum game have been widely used to study the evolution of cooperation and fairness among self-regarding individuals [5–25]. However, some social dilemmas involve multiple agents rather than two individuals, such as vaccination behavior [26]. The public goods game, the model most widely used for investigating N-person problems, has been extensively studied [27–37]. In a typical public goods game, each participant chooses either to cooperate (cooperator) by contributing amount c to the common pool or to defect (defector) without making any contribution. The total contributions are multiplied by an enhancement factor r ($1 < r < N$) and then distributed equally among all players regardless of their action. Therefore, defectors obtain a free ride on the back of contributions made by cooperators. Although the group can maximize its payoff if all have cooperated, the best strategy for a rational and selfish player is to defect, since every unit contribution invested is discounted as a return [38]. Thus, the social dilemma of what is best for an egoistic individual and what is best for the group arises [39]. This is also known as the tragedy of the commons [40].

To explain the conflict between the high prevalence of cooperation and the selfishness of individuals, specific mechanisms have been proposed, such as punishment, reputation, and voluntary participation [3,41–43]. Fehr and Gächte conducted a public goods experiment with and without punishment and found that apparent willingness to punish constitutes a credible threat for potential free riders and causes a large increase in cooperation level: very high or even full cooperation can be achieved and maintained under the punishment condition, whereas the same subjects converge towards

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full defection in the absence of punishment [3]. Milinski et al. found that cooperation in a public goods game pays off after introduction of a reputation mechanism in the game [41]. Hauert et al. verified that optional participation can foil defectors and overcome the social dilemma [42]. Using the notion of 'loners', they found that neither altruistic cooperators nor selfish defectors can stably dominate the population. Hence, the system usually evolves to an unending cycle of adjustments (a Red Queen type of evolution) rather than to an equilibrium.

Here we propose a recommendation mechanism based on group quality to investigate the emergence and maintenance of cooperation in a spatial public goods game. No punishment is applied and participation is compulsory. In real social systems, renowned universities, academic institutes, and successful companies are usually willing to recommend their members to other institutions or companies for further studies or to improve their skills to increase their success. Motivated by this observation, we assume that a certain individual in a group of good quality can be recommended for establishment of a new relationship with another, randomly drawn individual within a certain range in a different group. Using Monte Carlo simulation (MCS), we find that cooperation can flourish.

2. The model

In our co-evolutionary model, the populations are structured according to a square lattice (with a von Neumann neighborhood and periodic boundary conditions), an ER graph and a Barabási–Albert network [44]. The Barabási–Albert network is generated by a preferential attachment process starting with a small ring including m_0 nodes ($m_0 = 3$ here). At each time step, a new node with m links (to obtain the same average degree as the square lattice, we set $m = 2$) is added to the instantaneous network. The probability that each link is connected to one of the existing nodes is proportional to its current degree. Duplicate edges are prohibited. To generate the ER graph, we use a total of M links to pair the N nodes at random. To generate a network with the same average degree as the square lattice, we set $M = 2N$.

The vertices of the network denote individuals and the links characterize the partnerships between individuals. We define individual i and his neighbors as the group i. Initially, each individual is designated as a cooperator or a defector with equal probability. The game is iterated forward according to a sequential procedure consisting of recommendation, game playing and strategy updating. In the recommendation stage, each group is given an opportunity, according to a random sequence to recommend one of its members for establishment of a new relationship with an individual outside this group. Specifically, individual i belonging to group i may be recommended for connection to a randomly chosen member, not a neighbor of i, within a limited range d_0, which is the distance between these two individuals. Whether the recommended individual i will be accepted depends on the quality of group i, denoted by $Q_i = \frac{N_{ic}}{N_i}$, where N_i and N_{ic} are the number of members and of cooperators, respectively, in group i. If Q_i exceeds the threshold value Q_c, i is accepted and thus the recommendation event succeeds. Otherwise, i is declined and the recommendation fails. For simplicity, we assume that at most one individual in a group can be recommended. Thus, once an individual is recommended, his neighbors can no longer be recommended. This can be considered as competition for opportunities or resources. It also potentially prevents collapse of our system as a result of endless and duplicate recommendation. However, an individual is allowed to establish relationship with many individuals who are recommended by different groups. In each generation, Q_i is recalculated. If group i has recommended in previous generations and $Q_i < Q_c$, which implies that its quality has deteriorated, then the link created through recommendation will be detached, whereas if $Q_i \geq Q_c$ the link will be kept. Therefore, the relationship established through recommendation is not constant, but is dynamic. Its viability closely depends on the quality of the recommending group. Double and self connections are forbidden.

For interaction, individual x engages in $k_x + 1$ public goods games centered on his k_x neighbors and himself and accumulates a payoff. The payoff obtained by individual x from the game centered on his neighbor individual u is given by

$$p_{x,u} = \frac{r}{k_u + 1} \sum_{z \in \Omega_u} \frac{s_z}{k_z + 1}, \tag{1}$$

where Ω_u denotes the neighborhood set of u and himself, r is the enhancement factor, and s_z is the strategy of z. If z is a cooperator ($s_z = 1$), z allocates his total money, $c = 1$, equally to all public goods games he participates in. The payoff for each public goods game is evenly distributed among participants. The total payoff x collects can be evaluated as

$$P_x = \sum_{u \in \Omega_x} p_{x,u} - s_x. \tag{2}$$

Finally, individual x adopts the strategy of individual y who is randomly picked from all his neighbors with probability given by the Fermi function

$$W(s_y \to s_x) = \frac{1}{1 + \exp[-\beta(P_y - P_x)]}, \tag{3}$$

where β is the selection intensity ($\beta \to 0$ leads to random drift while $\beta \to \infty$ leads to deterministic dynamics). Following common practice [45], we set $\beta = 10$ here. In a full time step, each individual has a chance to imitate the behavior of his neighbors.