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Lapse of time effects on tax evasion in an agent-based econophysics model

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ABSTRACT

We investigate an inhomogeneous Ising model in the context of tax evasion dynamics where different types of agents are parameterized via local temperatures and magnetic fields. In particular, we analyze the impact of lapse of time effects (i.e. backauditing) and endogenously determined penalty rates on tax compliance. Both features contribute to a microfoundation of agent-based econophysics models of tax evasion.

1. Introduction

One of the first approaches to theoretically account for tax compliance was given by Allingham and Sandmo [1], which incorporates tax rates, potential penalties, and audit probabilities as basic parameters in order to evaluate the behavior of expected utility maximizing tax payers. However, it was realized early on that one of the major shortcomings of the Allingham and Sandmo theory is the prediction of much too low levels of tax compliance, as actually observed in industrialized nations. It is believed that these shortcomings are partially due to the insufficient consideration of interaction dynamics among the actors of the tax compliance game, i.e., tax payers, tax advisors, tax authorities, and tax law makers. Another reason might be that many analyses have not incorporated lapse of time effects, i.e. the situation that tax agencies perform an assessment of non-compliant tax payers which goes back in time to previous periods (backauditing). Although Allingham and Sandmo already considered backauditing (or lapse of time effects) as a possible extension of their basic theory, the issue of backauditing has been largely neglected in the literature. Notable exceptions are Alm et al. [2], Antunes et al. [3], and Hokamp and Pickhardt [4].

It is for these reasons that agent-based models have been set up as a comparatively new tool for analyzing tax compliance issues. In fact, an essential feature of any agent-based model is the direct non-market-based interaction among agents, which is combined with some process that allows for changes in individual behavior patterns. Therefore, agent-based tax evasion models may be categorized according to the features of this individual interaction process. In econophysics models, this process is commonly described within the Ising model [5], where examples include Zaklan et al. [6,7], Lima and Zaklan [8], and Lima [9].

In contrast, if the interaction process is driven by parameter changes that induce behavioral changes via a utility function and (or) by stochastic processes that do not have physical roots, these models belong to the economics domain. Examples

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include Mittone and Patelli [10], Davis et al. [11], Bloomquist [12,13], Korobow et al. [14], Antunes et al. [3], Szabó et al. [15], Meder et al. [16], and Hokamp and Pickhardt [4], of which some are summarized by Bloomquist [13] and Pickhardt and Seibold [17].

In all agent-based tax evasion models the actual patterns and levels of tax evasion depend on two additional factors: the network structure of society and the tax enforcement mechanism. The network structure is implemented by alternative lattice types, and tax enforcement consists of the two economic standard parameters audit probability and penalty rate.

In the present paper, we study the effect of alternative backauditing schemes on tax evasion within an multi-agent econophysics model. Moreover, backauditing also enables us to incorporate endogenously determined penalty rates. Both features, backauditing and endogenous penalties, contribute to the microfoundation of econophysics models and, therefore, allow for a more realistic modeling of tax compliance behavior within this modeling frame.

We present our model and formalism in Section 2. Results are analyzed in Section 3, where we discuss first the case of homogeneous societies in Section 3.1, which is then compared with the tax evasion of multi-agent societies in Section 3.2. We conclude our considerations in Section 4.

2. Model and formalism

Our considerations are based on the Ising model, described by the following energy:

$$H = -\sum_{ij} J_{ij} S_i S_j - \sum_i B_i S_i, \tag{1}$$

where J_{ij} describes the coupling of Ising variables (spins) $S_i = -1, 1$ between lattice sites R_i and R_j. In the present context, $S_i = 1$ is interpreted as a compliant tax payer and $S_i = -1$ as a non-compliant one. We implement the model on a two-dimensional 1000×1000 square lattice with nearest-neighbor interactions $J_{ij} \equiv J$ for $|R_i - R_j| = 1$ (lattice constant $a \equiv 1$). We expect that our results are robust with regard to variations of the network structure, in analogy to the investigations of Zaklan et al. [6]. Eq. (1) also contains the coupling of the spins to a local magnetic field B_i, which, together with a local temperature T_i, distinguishes the behaviorally different types of agent. Concerning the latter, we assume that it is imposed by the coupling of each lattice site i to a heat-bath with temperature T_i [18] to establish local thermal equilibrium. We then use the heat-bath algorithm [see [19]] in order to evaluate statistical averages of the model. The probability for a spin at lattice site i to take the values $S_i = \pm 1$ is given by

$$p_i(S_i) = \frac{1}{1 + \exp\{-[E(-S_i) - E(S_i)]/T_i\}}, \tag{2}$$

and $E(-S_i) - E(S_i)$ is the energy change for a spin-flip at site i. Upon picking a random number $0 \le r \le 1$, the spin takes the value $S_i = 1$ when $r < p_i(S_i = 1)$ and $S_i = -1$ otherwise. One time step then corresponds to a complete sweep through the lattice. We note that a generalization of the model with regard to the incorporation of non-equilibrium dynamics has been recently proposed in Refs. [9,20,21].

Following Ref. [4], we consider societies which are composed of the following four types of agent. (i) *Selfish a-type agents*, which take advantage from non-compliance and, thus, are characterized by $B_i/T_i < 0$ and $|B_i| \gg J$. (ii) *Copying b-type agents*, which copy the tax behavior of their social environment or neighborhood. This can be modeled by $B_i \ll J$ and $J/T_i \gtrsim 1$. (iii) *Ethical c-type agents*, which are practically always compliant and which are parameterized by $B_i/T_i > 0$ and $|B_i| \gg J$. (iv) *Random d-type agents* which are in principle like *c*-type agents, but due to some confusion caused by tax law complexity act by chance within a certain range. We implement this behavior by $B_i \ll J$ and $J/T_i \ll 1$. Here and in the following all parameters are measured with respect to $J \equiv 1$. Note that with regard to the previous definitions the analysis in Ref. [7] corresponds to homogeneous societies of b-type and d-type agents. Moreover, the model in Ref. [8] studies homogeneous societies of a-type and c-type agents. Our aim instead is the investigation of heterogeneous societies and the influence of different backaudit schemes and endogenously determined penalty rates on the extent of tax evasion.

We consider first the case where the detection of an evading agent in the current period enforces its compliance over the following h time steps or periods. Note that in econophysics models of tax evasion h is regarded as the penalty rate. The aforementioned procedure has been invoked in Refs. [6,7,9,17] and also implemented in a randomized variant in Ref. [8]. In Ref. [8], h has been interpreted as the time over which an agent is ashamed and feels guilty about his/her behavior after detection. An alternative interpretation would be definite audits by the tax authorities over h time steps after an agent has been detected for the first time, a scheme that is known as conditional future auditing (Alm et al. [2]). In addition, we study the situation where an audit may also allow for screening the agent's tax declaration over several years in the past (backaudit). The basic idea is that a backaudit is only performed by the tax authorities upon reasonable initial suspicion, which in our model corresponds to tax evasion in the current period. In the literature, this scheme is known as conditional backauditing (see Alm et al. [2]). We therefore introduce a probability p_a with which an audit is performed at a given lattice site (agent). If tax evasion is detected in the current period, the backaudit comprises also an inspection of the preceding b_p time steps. Denote with n_e the number of time steps over which the agent was evading during the backaudited periods plus the current period. Then the number of future periods k, over which the agent is forced to be compliant, is set to $k = n_e * h$. Thus, the penalty rate k is now endogenously determined via the parameter n_e. Note, however, that the above limit of a fixed number of enforced future compliance periods h is recovered in the limit of no backaudit $b_p = 0$.