Physica A 392 (2013) 2248–2256

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Opinion dynamics in networks with heterogeneous confidence and influence

Haili Liang, Yiping Yang, Xiaofan Wang *

Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai, 200240, People's Republic of China

A R T I C L E I N F O

Article history:
Received 27 May 2012
Received in revised form 18 November 2012
Available online 9 January 2013

Keywords:
Multi-agent systems
Opinion dynamics
Power law distribution
Influence radius

A B S T R A C T

We propose a discrete-time model of opinion dynamics. The neighborhood relationship is decided by confidence radius and influence radius of each agent. We investigate the influence of heterogeneity in confidence/influence distribution on the behavior of the network. The simulations suggest that the heterogeneity of single confidence or influence networks can promote the opinions to achieve consensus. It is shown that the heterogeneous influence radius systems converge in fewer time steps and more often in finite time than the heterogeneous confidence radius systems. We find that heterogeneity does not always promote consensus, and there is an optimal heterogeneity so that the relative size of the largest consensus cluster reaches maximum in heterogeneous confidence and influence networks.

1. Introduction

The analysis of multi-agent systems received an increasing attention in recent years. Given a set of autonomous agents, such as sensors, bees, cars, nodes of a communication network and robots, they start from different initial values, and interact according to some rules [1–6]. The interactions between agents are described by an interconnection topology. When we narrow down our interest to social networks and consider the evolution of opinions or beliefs, we are facing opinion dynamics or social learning problem. Any given agent observes the opinions through communication with a small subset of society, consisting of her friends, her coworkers, her family members and so on. The small subset of society is usually refer to as the social network of the given agent. DeGroot presented a model in which the group might reach a consensus and form a common subjective probability distribution for an unknown parameter by taking a weighted average of other agents' opinions [1]. In Vicsek's swarming model, all agents had the same speed with different headings [7]. But they could update their headings by averaging the headings of neighbors. The simulation results showed all agents might eventually move in the same direction without centralized coordination. Jadbabaie et al. [8] analyzed the linearized heading equations based on the nearest neighbor rules and proved that if the dynamical neighbor graphs are jointly connected in a certain "uniform" way, then the system will synchronize.

Structure of connection in social networks, in other words, neighborhood relationship has great influence on opinion dynamics model based on the nearest neighbor rules. If an agent only trusts those, whose opinions are close to its own, it can be considered that these agents have bounded confidence. Under this assumption, the communication structure changes dynamically with the opinions. If the confidence radii for all agents are equal, the model is homogeneous, otherwise it is heterogeneous. A heterogeneous opinion dynamics model will always be superior to a homogeneous model in the sense that communications need not to be symmetrical. The two most cited bounded confidence models are Hegselmann–Krause (HK) [2] and Deffuant–Weisbuch (DW) [9]. Each of them can be further classified into heterogeneous and homogeneous

* Corresponding author. Fax: +86 21 62932344.
E-mail addresses: lianghaili1016@gmail.com (H. Liang), xfwang@sjtu.edu.cn (X. Wang).

0378-4371/$ – see front matter
doi:10.1016/j.physa.2013.01.008

models, according to whether the confidence bounds are uniform or agent-dependent. The homogeneous models have been studied in various literatures. The basic HK and DW models have been extended in Refs. [10–12], in which it is proved that the homogeneous HK model reaches a steady state in finite time. In Ben-Naim's paper [13], each agent is initially assigned an opinion from some specified distribution. Their agents' radii are the same 1. This model is essentially identical to that of DW model. The system reaches a steady state with a finite number of isolated, non-interacting opinion clusters. As the initial opinion range increases, the number of such parties undergoes a periodic sequence of bifurcations. Blondel et al. [10] proposed that the confidence radii for all agents are equal to 1. They showed equilibrium inter-cluster distances are usually larger than 1, and typically close to 2. Ceragioli et al. [14] analyzed a continuous-time version of HK model of opinion dynamics with bounded confidence. They proved existence and completeness of solutions, and asymptotical convergence to a "clusterization" of opinions. The heterogeneous HK model was studied by Lorenz, who reformulated the HK model into an interactive Markov chain and analyzed the effects of heterogeneous confidence bounds. The confidence radii of these agents are generated according to the uniform distribution.

The previous studies rarely consider the impact of different bounded confidence and influence for the discrete models of opinion evolution. The communication relationship in the most existing models of bounded confidence are only decided by opinions of the agents. This assumption is too strict when we consider communications in the real world, where two people cannot communicate with each other if their geographic locations or statuses in society do not allow, even through they have exactly the same opinion. Therefore, it is necessary to investigate the heterogeneity of bounded confidence and influence in discrete-time cases. Yang et al. [15] considered Vicsek's swarming model in a heterogeneous influence network. The influencing capability of each agent is represented by its influencing radius. The main conclusion is that agreement among a few powerful leaders in a heterogeneous influence network is the key to achieving global direction consensus for the whole system. Inspired by the work of Yang et al. [15], we proposed a opinion dynamics model with heterogeneous agents and take into account each agent's confidence/influence radius.

The distribution of each agent's confidence/influence radius does not follow a uniform distribution but follows a power law distribution $p(r) \sim r^{-\alpha}$ with exponents $\alpha \in (2, +\infty)$. The aim of this paper is to investigate the influence of heterogeneity in confidence/influence distribution on the behavior of the network. We analyze the model of opinion dynamics by considering three cases in confidence and influence networks. For any initial condition, all opinions converge to clusters and keep the value after a finite time. The simulations suggest that the heterogeneity of single confidence and influence networks can promote the opinions to achieve consensus. It is observed that the heterogeneous influence radius systems converge in fewer time steps and more often in finite time than the heterogeneous confidence radius systems. Numerical simulations illustrate that heterogeneity does not always promote consensus, and there is an optimal heterogeneity so that the size of the largest consensus cluster reaches the maximum in heterogeneous confidence and influence networks. Confidence radius plays a key role in determining the number of clusters in heterogeneous confidence and influence networks.

This paper is organized as follows. Section 2 describes the discrete-time opinion dynamics model. Section 3 investigates the opinion dynamics model with heterogeneous confidence radius and large enough influence radius. Section 4 shows the model of opinion dynamics with heterogeneous influence radius and large enough confidence radius. Section 5 considers the influence of heterogeneity in confidence and influence distribution on the behavior of the network. Section 6 concludes this paper and suggests directions for future work.

2. The discrete-time model

Consider a system consisting of n agents. And assume that at each discrete time k, agent i keeps a real number representing his/her opinion $x_i(k)$. These values are synchronously updated according to

$$x_i(k+1) = x_i(k) + \frac{1}{|N_i(k)|} \sum_{j \in N_i(k)} (x_j(k) - x_i(k)), \tag{1}$$

where $N_i(k)$ is the set of agent i's neighbors at time k. And $|N_i(k)|$ is the cardinality of $N_i(k)$.

r_i and R_i are agent i's confidence radius and influence radius, respectively. The opinion $x_i(k)$ of the agent i is affected by the agent j' opinion $x_j(k)$ if $|x_i(k) - x_j(k)| \le \min\{r_i, R_j\}$, $r_i > 0$, $R_j > 0$ in bounded confidence and influence model. That is

$$N_i(k) = \{j \in \{1, 2, \dots, n\} : |x_i(k) - x_j(k)| \le \min\{r_i, R_j\}\}.$$

The bounded confidence radius r_i or bounded influence radius R_j is randomly chosen according to power law distribution $p(r) \sim r^{-\alpha}$ with a scaling exponent $\alpha \in (2, +\infty)$. In other words, each agent has some self-confidence and self-influence. The initial opinion value $x_i(0)$ is randomly generated and the range is [0, 10]. $p(r) = Cr^{-\alpha}$ is a probability density function. The confidence/influence radii distributions are power-law distributions, thus the radii are different according to various exponents $\alpha \in (2, +\infty)$.

Newman's transformation method can be used to get a randomly power law distributed real number in the range $r_{\min} \le r < \infty$ with the exponent α [16]. If there is a random real number Ψ uniformly distributed in the range $0 \le \Psi < 1$, the random power law distributed real number is: $r = r_{\min} \left(\frac{1}{1-\Psi}\right)^{\frac{1}{\alpha-1}}$. And the average radius is: $\langle r \rangle = \frac{\alpha-1}{\alpha-2} r_{\min} (\alpha > 2)$.