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The influence of learning and updating speed on the growth of commercial websites

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ABSTRACT

In this paper, we study the competition model of commercial websites with learning and updating speed, and further analyze the influence of learning and updating speed on the growth of commercial websites from a nonlinear dynamics perspective. Using the center manifold theory and the normal form method, we give the explicit formulas determining the stability and periodic fluctuation of commercial sites. Numerical simulations reveal that sites periodically fluctuate as the speed of learning and updating crosses one threshold. The study provides reference and evidence for website operators to make decisions.

1. Introduction

Recent years have witnessed the prosperous development of e-commerce market. The report of monitoring data on the Chinese e-commerce market, which is issued by Chinese e-commerce Research Center in 2010, indicated that the number of B2B e-business transactions in China amounted to 9200, 21.3 percent greater than that over the same period of the last year. Meanwhile, the number of B2C, C2C and other non-mainstream e-business transactions also reached 15,800, 58.6 percent more than in 2009, and would be over 20,000 in 2011. The increasingly fierce competition among e-commerce websites is forcing their managers to explore the interior growth mechanism and causes of sites.

In recent years, the competition among websites is acquiring an increasingly important position. Maurer and Huberman [1] first proposed the competition model of websites based on the Lotka–Volterra competitive system in biology, which is described by

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \alpha_i x_i(\beta_i - x_i) - \sum_{j\neq i, j=1}^{n} \gamma_{ij} x_i x_j, \tag{1}$$

where x_i is the fraction of the population that is a customer of site i, α_i is the growth rate which measures the capacity of site i to grow, β_i is the maximum capacity which denotes the saturation value of x_i, and γ_{ij} is the competition rate between sites i and j. The greater γ_{ij} is, the more loss of users of site i caused by the existence of site j will be. $0 \le x_i \le 1, \alpha_i > 0, 0 < \beta_i \le 1, \gamma_{ij} \ge 0,\ i, j = 1, \ldots, n$.

After that, there has been an extensively academic literature about the modeling and competitive strategy of websites (for example, see Refs. [2–9]). Specifically, López and Sanjuan [2] defined the collaboration between sites as low competition

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and offered some interesting strategies for collaborating small sites to win. Wang and Wu [3] established one cooperative and competitive model and Jiang and Cheng [4] gave the complete strategic classification for this model in Ref. [3]. Xiao and Cao [5] presented a delayed competitive website model and studied the direction and stability of bifurcated periodic solutions by using the Hassard method. Ren et al. [6] analyzed the stability of the competitive model of e-commerce websites with market segment. Li and Zhu [7–9] explored the effect of competition and the characteristic of "rich gets richer" on the Internet economy and established the following model

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \alpha_i x_i(\beta_i - x_i) - \sum_{j\neq i, j=1}^{n} \gamma_{ij}(1 + \omega_{ij}(x_j - x_i))x_i x_j, \tag{2}$$

where $\omega_{ij}(x_j - x_i)$ describes the characteristic "rich gets richer" of e-commerce websites, ω_{ij} denotes the effect degree and $0 < \omega_{ij} < \min\{1, \gamma_{ij}\}$.

As a matter of fact, the speed of learning and updating is extremely significant for some young e-commerce websites. For example, Prosper.com, one Peer-to-Peer (P2P) lending website which is launched in 2006 in USA, holds the advantage position, and has become the American largest peer-to-peer lending marketplace by constant and quick learning and updating in the business models, service strategies, transmission mechanisms, etc. But unfortunately, the studies mentioned above hardly discuss the influence of learning and updating speed on the growth of commercial websites.

As the complementarity and extension, in the present paper, we will denote time delay as the speed of learning and updating of e-commerce websites, and devote our attention to the following competition model

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \alpha_i x_i(t)(\beta_i - x_i(t - \tau_i)) - \sum_{j\neq i, j=1}^{n} \gamma_{ij}(1 + \omega_{ij}(x_j(t) - x_i(t)))x_i(t)x_j(t), \tag{3}$$

where $\tau_i > 0$ represents the learning and updating speed. Obviously, for site i, the smaller τ_i, the bigger advantage.

Due to the complexity of multi-dimensional nonlinear competitive system with time delays, sometimes, we have to make some restrictive assumptions to obtain some valuable information. Here, we mainly focus on the competition between two e-commerce websites. One is powerful, the other is small, and their strengths are nip and tuck ($\alpha_i = \alpha, \beta_i = \beta, \omega_{ij} = \omega, \gamma_{ij} = \gamma$). Namely,

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = \alpha x_1(t)(\beta - x_1(t - \tau)) - \gamma(1 + \omega(x_2(t) - x_1(t)))x_1(t)x_2(t), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = \alpha x_2(t)(\beta - x_2(t - \tau)) - \gamma(1 + \omega(x_1(t) - x_2(t)))x_1(t)x_2(t). \end{cases} \tag{4}$$

The rest of this paper is organized as follows. In Section 2, the effect of learning and updating speed on the growth of e-commerce websites is studied. In Section 3, the explicit formulas determining the developing stability and periodic fluctuation of sites are obtained by applying the center manifold theory and the normal form method. In Section 4, some numerical simulations are performed to illustrate the analytical results. Finally, we conclude the work in Section 5.

2. The influence of learning and updating speed on the growth of commercial websites

Definition 1 (*Ren et al. [6]*). In System (3), we say website j strongly competes against website i, if $\frac{\alpha_i}{\gamma_{ij}} < 1$. Otherwise, if $\frac{\alpha_i}{\gamma_{ij}} > 1$, we say website j weakly competes against website i.

Definition 2 (*Ren et al. [6]*). In System (3), we say a site is powerful if it has the maximum fraction of customers at the initial time. Other sites are called small sites. We also say a site is monopoly, if a site occupies all market shares and the others tend to extinction.

Clearly, when the process of learning and updating is neglected, namely $\tau = 0$, System (4) exists in the following steady states:

$$P_0(0, 0), \qquad P_1(\beta, 0), \qquad P_2(0, \beta), \qquad P_3\left(\frac{\alpha\beta}{\alpha+\gamma}, \frac{\alpha\beta}{\alpha+\gamma}\right), \qquad P_4(u, v), \qquad P_5(v, u),$$

where $u = \frac{\alpha-\gamma+\sqrt{-\alpha^2+\gamma^2+2\alpha\beta\gamma\omega}}{2\gamma\omega}$, $v = \frac{\alpha-\gamma-\sqrt{-\alpha^2+\gamma^2+2\alpha\beta\gamma\omega}}{2\gamma\omega}$.

By straightforward computation, when $\frac{\alpha}{\beta\omega+\sqrt{1+\beta^2\omega^2}} \le \gamma \le \frac{\alpha}{1+\beta\omega}$, the equilibria P_4, P_5 will be meaningful (u, v are non-negative).

For the stability of above equilibria, Ref. [7] gave the following conclusions.

Lemma 1. (1) *For all $\gamma > 0$, the equilibrium $P_0(0, 0)$ is unstable.*
(2) *When $0 < \gamma < \frac{\alpha}{\beta\omega+\sqrt{1+\beta^2\omega^2}}$, the equilibria $P_1(\beta, 0), P_2(0, \beta)$ are unstable, and $P_3(\frac{\alpha\beta}{\alpha+\gamma}, \frac{\alpha\beta}{\alpha+\gamma})$ is stable.*