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Dependency of percolation critical exponents on the exponent of power law size distribution

Q1 S. Sadeghnejad [a,*], M. Masihi [b], P.R. King [c]

[a] Department of Chemical Engineering, Tarbiat Modares University, Islamic Republic of Iran
[b] Department of Chemical and Petroleum Engineering, Sharif University of Technology, Islamic Republic of Iran
[c] Department of Earth Science and Engineering, Imperial College, United Kingdom

HIGHLIGHTS

- The effect of power law size distribution on the percolation theory is investigated.
- Deviations of critical exponents from the universal values investigated numerically.
- Two different object shapes i.e., stick-shaped and square are considered.
- Studying the connectivity of systems with a very broad size distribution is improved.

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ABSTRACT

The standard percolation theory uses objects of the same size. Moreover, it has long been observed that the percolation properties of the systems with a finite distribution of sizes are controlled by an effective size and consequently, the universality of the percolation theory is still valid. In this study, the effect of power law size distribution on the critical exponents of the percolation theory of the two dimensional models is investigated. Two different object shapes i.e., stick-shaped and square are considered. These two shapes are the representative of the fractures in fracture reservoirs and the sandbodies in clastic reservoirs. The finite size scaling arguments are used for the connectivity to determine the dependency of the critical exponents on the power law exponent. In particular, the deviations of percolation exponents from their universal values as well as the connectivity behavior of such systems are investigated numerically. As a result, this extends the applicability of the conventional percolation approach to study the connectivity of systems with a very broad size distribution.

1. Introduction

There are many phenomena in physics, e.g., highly disordered media, whose effective behavior is dominated by the connectivity of the system and can be described well by the percolation theory [1]. The percolation theory was first introduced by Flory et al. [2] and Stockmayer [3] and later it was developed by Broadbent and Hammersley [4]. Since then, it has intensively been studied in physics literature (see Stauffer and Aarony [1] and references therein). Percolation theory has many applications from the spread of diseases [5,6] and forest fires [7] to polymer materials [8], porous media [9], and petroleum reservoirs [10–12].

* Corresponding author. Tel.: +98 9122865638.
E-mail addresses: s_sadeghnezhad@yahoo.com, sadeghnejad@mapsatech.com (S. Sadeghnejad), masihi@sharif.edu (M. Masihi), peter.king@imperial.ac.uk (P.R. King).

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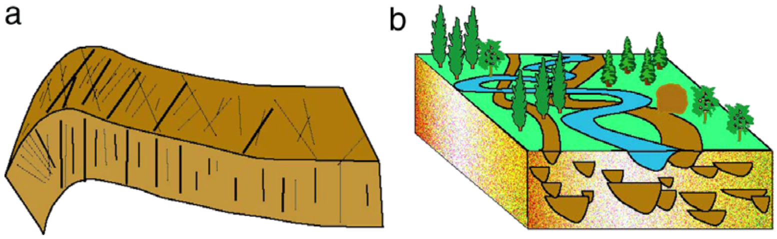

Fig. 1. Illustration of two systems (a) a fracture reservoir and (b) a channel reservoir, with long-range correlation.

The percolation theory is a mathematical model of the connectivity of randomly distributed objects in complex geometries. The global, geometrical, and physical properties of such a system are related to the density of objects randomly placed in a domain from which all outcomes can be predicted by simple transformations through some power laws with universal exponents [1]. In addition, these laws have been postulated that are independent of the detail of system, i.e., local geometries [1]. By universality in the classic percolation, one means that the power laws as well as the exponents are independent of the size and shape of objects in the system and they only depend on dimensionality of space (i.e., 2-D or 3-D space). The principle of universality is a result of the fact that the percolation transition (i.e., transition from the unconnected to the connected system) is a second order phase transition like the liquid–gas transition in thermodynamics [13]. This is a powerful concept that enables us to study the behavior of a very wide range of systems without needing to worry about the fine scale details. Examples for checking the universality concept and determination of the percolation threshold of various continuum models can be found elsewhere (e.g., Baker et al. [14]; Lee and Torquato [15]; Lin and Hu [16]; Lorenz and Ziff [17]; Xia and Thrope [18]).

The classic percolation theory assumes spatially uncorrelated systems filled by objects with the same size [1,19]. However, for many natural systems there is a long-range correlation and/or a broad size distribution. For example, connectivity of fracture networks with distribution of size of fractures (i.e., represented in this study by sticks) is an interesting example in geological systems (Fig. 1(a)). In these systems, the fractures with different sizes can intersect each other and consequently make a complicated fracture network. In petroleum reservoirs, these networks are hot spot regions that become the targets for drilling new wells in order to recover more hydrocarbons. Another attractive example is an underground reservoir composed of the lenticular porous clastic rocks (e.g. the channel reservoirs). These types of reservoirs consist of lenticular oil-bearing sands with a distribution of sizes deposited in an impermeable background (Fig. 1(b)). In such systems, the distribution of size of permeable objects plays an important role in the connectivity at the reservoir scale as well as the amount of recoverable oil.

It has been shown that the short-range correlation and/or scale limited size distributions do not affect the critical exponent of the percolation theory. Harter [20], for example, has shown that the percolation threshold in Markov chain random field with short range correlation decreases as the correlation scale increases but the critical exponents are unaffected (i.e., they belong to the same universality class as uncorrelated percolation). For the systems with long-range correlations and/or power law length distribution, on the other hand, it has been expected that the percolation behavior is drastically changed [21–29].

It is shown that the connectivity behavior of the system made of objects with a scale limited distribution of sizes (e.g., uniform and Gaussian distribution) is identical to the connectivity behavior of a system made of constant-size objects with an effective size defined based on an appropriate moment of objects' size distribution [30–33]. However, this hypothesis does not work on very broad size distributions such as power law distribution. In such a system, very large objects (e.g., an object in the scale of the system size) can be existed and consequently dominate the global connectivity of the system [34–36]. This means that the nature of the finite size scaling law and the universality property will change.

In this study, percolation properties of two systems made of (1) stick shaped objects (for example representing fractures in the fractured rock systems [37–39]) and (2) squared shape objects (for instance representing sandbodies in the clastic reservoirs [30,40–42]) with a power law size distribution are investigated.

2. Percolation theory

Continuum percolation models form from number of objects randomly distributed within the system, which can freely overlap on each other and consequently make the cluster of objects. The area/volume fraction of the system occupied by the objects is called the occupancy probability, p. As p increases the clusters grow in size and at the percolation threshold, p_c^∞, an infinite cluster (i.e., percolation cluster) appears. Percolation probability, P, (i.e., connectivity or connected fraction) is defined as the probability that any point in the system belongs to the percolation cluster. For infinite systems around the percolation threshold, p_c^∞, the following power laws apply:

$$P(p) \propto \left(p - p_c^\infty\right)^\beta \qquad (1)$$

$$\xi(p) \propto \left(p - p_c^\infty\right)^{-\upsilon} \qquad (2)$$

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