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Binary collision approximation for multi-decorated granular chains

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HIGHLIGHTS

- We study pulse propagation in tapered multi-decorated granular chains.
- The decorated chain is approximated by an effective undecorated one.
- The binary collision approximation is applied to the effective chain.
- We obtain the residence time of the pulse on each large grain.
- Our results are in good agreement with numerical results for the original chain.

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ABSTRACT

We study pulse propagation along decorated tapered granular chains without precompression. Our goal is to generalize the results obtained in our previous work, by analyzing a decorated chain with an arbitrary number of small grains between the large ones. Making use of an effective description, where the original decorated tapered chain is replaced by a non-decorated tapered chain with effective masses interacting via an effective potential, and applying the binary collision approximation, we calculate the residence time of the pulse on each effective large grain. We also present the comparison between the numerical integration of the equations of motion and our analytical predictions which show the agreement to be very good for the pulse velocity, albeit only qualitatively for the velocity of the grains.

1. Introduction

Pulse propagation in granular chains has attracted the attention of many scientists since it was first shown that an initial impulse imparted at one end of the chain could give rise to solitary waves [1,2]. Different types of chains have been studied since then. In particular, polydispersity is one of the characteristics of granular chains which has been considered in regular fashion, as in tapered [3–12], and/or randomly structured chains [1,13–16].

Recently [12] we studied pulse propagation in a mono-decorated tapered chain, that is, a chain of large grains with radii that decrease systematically, with a small grain between each pair of large grains. In this contribution, we are interested in pulse propagation in a decorated granular chain decorated with an arbitrary number of small grains between the large ones. In particular, we are interested in the velocity profiles and the residence time of the pulse as the initial disturbance travels along the chain. Our main purpose is to apply the binary collision approximation to describe pulse propagation along the chain. This theory has been successfully employed to analyze the pulse propagation through several types of chains,

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including monodisperse [17], tapered [11], *o*-rings with and without precompression [18], decorated [12] and randomly decorated [16], and *Y*-shaped chains [19,20]. However, as has already been pointed out in Ref. [12], the binary collision approximation cannot be directly applied to decorated chains because the energy and momentum transfer occur through several oscillations of the small grains instead of a single forward transfer, as assumed by the approximation. Therefore we proceed to obtain an effective chain composed of only large grains. These new large grains have effective masses and interact via effective potentials which we calculate here.

This paper is organized as follows. We describe the interaction model in Section 2. In Section 3 we obtain the effective potential, while in Section 4 we present the binary collision approximation for the effective chain. In both sections we show comparisons of the theoretical predictions with the numerical integration of the equations of motion. Finally, we present concluding remarks in Section 5.

2. Model

We consider chains of grains placed along a line so that they just touch their neighbors (there is no precompression), and all but the leftmost particle are at rest. The equation of motion for the k-th grain (except for the first and last grains) is

$$M_k \frac{\mathrm{d}^2 y_k}{\mathrm{d}\tau^2} = a r'_{k-1}(y_{k-1} - y_k)^{n-1}\theta(y_{k-1} - y_k) - a r'_k (y_k - y_{k+1})^{n-1}\theta(y_k - y_{k+1}). \tag{1}$$

For the first (last) grain, the first (second) term in the right-hand absent. M_k is the mass of grain k, y_k is its displacement from its initial position and a is a constant determined by Young's modulus and Poisson's ratio [21,22]. The Heaviside function $\theta(y)$ ensures that the elastic interaction between grains only exists if they are in contact. The constant r'_k is given by

$$r'_k = \left(\frac{2R'_k R'_{k+1}}{R'_k + R'_{k+1}}\right)^{1/2}, \tag{2}$$

where R'_k is the principal radius of curvature of the surface of grain k at the point of contact with grain $k+1$. The initial velocity V_1 of the leftmost particle ($k = 1$) provides the initial perturbation which will give rise to a propagating pulse [13]. For convenience, we introduce the dimensionless quantity

$$\alpha \equiv \left[\frac{M_1 V_1^2}{a(R'_1)^{n+1/2}}\right]^{1/n}, \tag{3}$$

and the rescaled quantities x_k, t, m_k, and R_k via the relations

$$y_k = R'_1 \alpha^{1/n} x_k, \qquad \tau = \frac{R'_1}{V_1}\alpha^{1/n} t, \qquad R'_k = R'_1 R_k, \qquad M_k = M_1 m_k, \tag{4}$$

so that the Eq. (1) can be rewritten as

$$m_k \ddot{x}_k = r_{k-1}(x_{k-1} - x_k)^{n-1}\theta(x_{k-1} - x_k) - r_k (x_k - x_{k+1})^{n-1}\theta(x_k - x_{k+1}). \tag{5}$$

A dot denotes a derivative with respect to t, and

$$r_k = \left(\frac{2R_k R_{k+1}}{R_k + R_{k+1}}\right)^{1/2}. \tag{6}$$

In the rescaled variables, the initial velocity is unity, i.e., $v_1(t = 0) = 1$, while the velocity of the k-th grain in the unscaled variables is simply V_1 times its velocity in the scaled variables.

3. Effective dynamics

When an initial impulse is imparted at one end of a tapered chain decorated with N small grains between each pair of large grains, energy and momentum travel along the chain. A large grain pushes the subsequent small grains which oscillate and transfer momentum and energy. Despite the fact that the dynamics of these oscillations is not easily described, it is possible to approximately describe the large grain dynamics by designing an effective chain composed of large grains with effective masses, interacting via effective potentials. In order to design the effective chain, we follow a procedure similar to that in Ref. [12] and write the position of the small grains as

$$x_k(t) = \bar{x}_k(t) + g_k(t), \tag{7}$$

where $\bar{x}_k(t)$ represents the equilibrium position of the small grains at any time and $g_k(t)$ is its deviation from $\bar{x}_k(t)$. Our main approximation is to assume that g_k is small enough to be neglected. The dynamics of a short chain composed of three large