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Coupling of two asymmetric exclusion processes with open boundaries

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HIGHLIGHTS

- We present a two-channel ASEP model with a general class of transition rates.
- We study the effect of horizontal transition rates on the steady-state properties.
- The effect of SSEP dynamics in both the lanes has been examined.
- We develop the phase diagram in the partially asymmetric coupling case.

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ABSTRACT

We study the dynamics of a coupled two-channel ASEP in which intra-channel transition rates are dependent on the configuration of neighboring channel. The binding constant k, which signifies the ratio of inter-channel transition rates, is introduced and the symmetric and asymmetric coupling conditions are analyzed for different values of k. The vertical cluster mean-field theory is used to study the system behavior exactly in strong coupling conditions and approximately in intermediate coupling conditions. Additionally, the consequences of particular dynamics such as totally asymmetric simple exclusion process (TASEP), partially asymmetric simple exclusion process (PASEP) and symmetric simple exclusion process (SSEP) in either one or both channels are investigated. It is found that the transition rates have a significant influence on both the qualitative and quantitative nature of the phase diagrams. The mathematical computation shows how the number of phases varies from 3 via 6 to 7 under different environments. Interestingly, in the fully asymmetric coupling case, the results are found to be independent of the magnitude of non-zero vertical transition rate. Our theoretical arguments are in well agreement with extensively performed Monte-Carlo simulation results.

1. Introduction

Nature presents plentiful examples of system driven far away from equilibrium. In contrast to the systems in equilibrium, we do not have any general analytical framework which describes non-equilibrium systems in a unified manner. The most important property of a non-equilibrium system is the existence of non-zero particle current in its steady-state. To characterize many-particle systems, it is very important to gain insight about the non-equilibrium steady-states. Driven lattice gas (DLG) models are one kind of discrete models which are widely used to study such systems. In DLG, particles interact with their nearest neighbors and hop in a preferred direction. The asymmetric simple exclusion process (ASEP) is a specific example of DLG and is considered to be the simplest stochastic model for transport phenomena in which particles move along the lattice obeying hard-core exclusion principle with certain pre-assigned rules. The usefulness of this approach can

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be seen from its suitability to describe successfully various physical, chemical and biological processes such as kinetics of bio-polymerization [1], protein synthesis [2,3], dynamics of motor proteins [4], diffusion through membrane [5], gel electrophoresis [6], vehicular traffic [7,8] and modeling of ant-trails [9], etc. Despite their simplicity, these models are competent to efficiently explain some complex non-equilibrium phenomena such as boundary-induced phase transitions [10–13], phase separation [14], spontaneous symmetry breaking [15] and localized shocks [16,17], etc.

The models based on the ASEP approach have been widely studied and solved exactly for some particular cases using various analytical treatments such as the Bethe ansatz method, mean-field theory, re-normalization group and oscillator algebra [18–20], etc. In the literature, several extensions of ASEPs in the form of introducing local inhomogeneities in the lattice [21], multi-species models [22–24], arbitrary particle size [25] and coupling with Langmuir kinetics [26,27] etc. have been discussed. Single-channel ASEPs have been studied comprehensively and are found to exhibit the aforementioned non-equilibrium phenomena. There exist a large number of real processes such as vehicular traffic, macroscopic clustering phenomena, motor protein dynamics and various systems of oppositely moving particles [7,8,28–32] which comprise particles moving in more than one channel. Therefore, in order to understand the dynamical aspect of many-particle systems more adequately, it becomes important to analyze multi-channel ASEPs. In the last decade, several investigations have been carried out on multi-channel ASEPs. Pronina and Kolomeisky [33] introduced vertical cluster mean-field theory to analyze the role of coupling strength between two channels. Later, the use of this theory made contributions towards the investigation of weakly as well as strongly coupled two-channel systems [34–36]. In a totally asymmetric simple exclusion process (TASEP) two-channel model, the possible existence of seven distinct steady-state phases has been shown in the case when lane changing is completely biased in one direction [34]. Recently, similar consequences have been found by Shi et al. [35] for a two-channel partially asymmetric simple exclusion process (PASEP). Tsekouras and Kolomeisky [36] investigated the role of coupling rates in a coupled TASEP and symmetric simple exclusion process (SSEP). Additionally, several other aspects like symmetry breaking, phase coexistence, phase separation with bidirectional transport and shock formation have been explored in multi-channel ASEP models [37–47]. Recently, boundary layer analysis has been performed for a two-channel ASEP by Yadav et al. [48] using the fixed point method.

In spite of the substantial work done on multi-channel ASEPs, the steady-state dynamical behavior of such systems is theoretically analyzed up to a limited range of parameters only. The conditions of asymmetric coupling rates between two channels have been studied only for some specific values of vertical transition rates [35,36]. Keeping in mind the diversity in magnitude of lane-changing rates in real multi-channel systems, we are motivated to extend the previous work to determine the steady-state solutions which are valid for all possible values of inter-channel as well as intra-channel transition rates. In certain physical processes such as multi-lane vehicular traffic, the hopping rate of a vehicle in its channel is indirectly influenced by the presence of a vehicle in its neighboring channel. Incorporating this idea into the dynamical rules of a two-channel ASEP, one can understand the stationary properties of the system more convincingly. The purpose of this paper is to provide a complete description of the dynamics of a general two-channel ASEP which is more practical because of the dependency of intra-channel transition rates on the configuration of the neighboring lane. In our opinion, we have investigated all the possible steady-state solutions for symmetric and asymmetric coupling in the limits of both strong and intermediate cases separately. Monte-Carlo simulations are also performed to justify the theoretical predictions.

The paper is organized as follows. In Section 2, we define the two-channel ASEP model and its governing dynamical rules. The theoretical description using mean-field theory and analysis of the stationary properties of the model are covered in Section 3. The phase diagrams as well as simulation results are discussed under different conditions originating from the values taken up by various parameters in Section 4. In the concluding section, we summarize the results and future prospectives of our work.

2. The model and dynamical rules

The model consists of indistinguishable particles distributed under the exclusion principle along two parallel lattice channels each of length L. Particles can move horizontally, i.e. forward and backward along the channel and inter-channel transitions are also allowed. A site is randomly selected from the two-lane system at each time step and any kind of transition is possible only when the target site is vacant. Particles can enter with a rate α in both the lanes when the entrance site is empty. Particle movements in the bulk of the system are characterized by following dynamical rules (Fig. 1). The vertical transition rates are w_1 and w_2 for lane-1 and lane-2, respectively. The forward and backward hopping rates of a particle on ith lattice site in lane-1 depends on the occupancy state of ith site in lane-2 and vice-versa. Particles in jth lane ($j = 1, 2$) move towards right with a rate p_j and towards left with a rate q_j when the corresponding site on the neighboring lane is occupied; otherwise forward and backward hopping rates are taken as $p_j(1 - w_j)$ and $q_j(1 - w_j)$, respectively. Such a rule is defined in order to ensure that the full transition rate of the particle from a site to go backward/forward/up/down is always 1. The rate with which particles can leave the lane is β if the exit site on the other lane is occupied otherwise the exit rate from jth lane becomes $\beta(1 - w_j)$.

The proposed model can be viewed as a generalization of models given by Pronina and Kolomeisky [33,34], Tsekouras and Kolomeisky [36] and Shi et al. [35]. The dynamical rules chosen by us are more realistic. Taking the example of vehicular traffic, this can be understood as whenever a vehicle can change its lane, the hopping rate of the vehicle along its channel will be reduced. The physical relevance of this model lies in its capability to describe transport phenomena with a wider perspective on the transition rates.