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# Q2 Role of population density and increasing neighborhood in the evolution of cooperation on diluted lattices

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## HIGHLIGHTS

- We investigate the influence of population density on the evolution of cooperation.
- We analyze the role of increasing neighborhood size in the promotion of cooperation.
- The optimal population density exists in the pair-wise game model.
- The intermediate neighborhood size is fittest for the evolution of cooperation.

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## ABSTRACT

We investigate the evolution of cooperative behaviors with increasing neighborhood size on diluted lattices. For three typical pairwise game models which include prisoner's dilemma, snowdrift and stag hunt games, all numerical results indicate that cooperation can persist or emerge around the optimal population density which is dictated by the percolation threshold on the square lattice. Meanwhile, the neighborhood size determines the interaction ranges of focal players and then dominates the percolation threshold, and extensive numerical simulations demonstrate that the intermediate neighborhood size is the most beneficial to the evolution of cooperation in the current lattice setup. The current findings can help to deeply understand the sustenance and emergence of collective cooperation in many natural, social and economic systems.



## 1. Introduction

In recent years, understanding collective cooperation has become an interdisciplinary challenging task within the scientific communities [1,2] which include biology, mathematics, physics, engineering and social sciences *etc.* Among them, evolutionary game theory has laid out a solid foundation for us to analyze the evolution of cooperation [3–5]. Starting from this theoretical framework, various microscopic mechanisms are presented to explore the potential origins of cooperation between unrelated individuals. For example, kin selection [6], direct or indirect reciprocity [7,8] and group selection [9] have been found to be effective means to promote cooperative behaviors. In addition, some extra methods are also put forward to interpret the ubiquity of cooperation, such as reward and punishment [10], voluntary participation [11,12], individual reputation [13], conditional strategy [14], age structure [15–17], strategy imitation [18] or success-driven distribution [19], Q3

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and so on. Meanwhile, various game techniques are also utilized as theoretical tools to analyze the price-determining rule under economical and financial environments [20,21].

However, extensive works focus on the role of game behaviors or microscopic mechanisms in the evolution of cooperation, and the interaction pattern among individuals or the macroscopic structure in the whole population is neglected, and is often hypothesized to be well-mixed. In particular, beyond the well-mixing hypothesis, Nowak and May [22] seminally investigated the game behavior and spatial chaos among a population where players are allocated on the square lattice. They found that the spatial structure of the lattice helps the cooperators to build cooperative clusters, and then protects them from being invaded by defectors. Inspired by this pioneering work, many authors integrate large quantities of mechanisms into the spatial lattices to discuss the evolution of cooperative behaviors in a colony, such as environmental influences [23–25], weight distribution [26–29], individual mobility [30–34], learning and teaching activities [35–37], memory effect [38], aspiring to the fittest payoff [39,40], noise-induced enhancement [41], and so on. Nevertheless, regular topology is still far from the topological structure embedded in many real-world systems which often exhibit complex topology properties comprising the small-world effect and scale free degree distribution [42]. Exploring the cooperative behaviors on complex networks or graphs has become an active topic in the physics community [3–5], and Santos et al. [43,44] indicated that the high degree nodes (i.e., hub nodes) in complex networks have adaptive advantages for cooperators and organize into giant cooperative clusters against the employment of defectors, leading to a dramatic going up in the level of cooperation. Especially in heterogeneous scale-free networks, a single cluster containing the most connected individuals is always formed to favor the cooperators in the prisoner's dilemma game [45]. Furthermore, the topology may adapt as the cooperation evolves, and thus coevolutionary behavior between the network and game cooperation also attracts the attention of researchers in many scientific fields [46].

In the typical framework of evolutionary game theory, the pairwise interaction game model is often used to investigate the cooperative behaviors under natural and social environments. Generally, in a simple pairwise game model, individuals often take two distinct strategies, to cooperate ($C$) or defect ($D$), and simultaneously make a decision during the game processes. Two players will both receive the reward ($R$) if they decide to cooperate mutually, but suffer from the punishment ($P$) due to mutual defection. The cooperator will get the sucker's ($S$) payoff and their opponent will obtain the temptation ($T$) to defect if two players choose different strategies. Depending on the payoff ranking order [1,2], various pairwise models can be implemented, such as the prisoner's dilemma game (PDG) if $T > R > P > S$, the snowdrift game (SDG) if $T > R > S > P$ and the stag hunt game (SHG) if $R > T > P > S$. Most previous works often assume that each site on a square lattice or complex network can contain at least one player, but in some cases the node position cannot be held by any player. Among them, Vainstein and Arenzon [47] first studied the robustness of cooperation in heterogeneous ecosystems in a spatial PDG model by considering site diluted lattices, and they demonstrated that the fraction of cooperators is enhanced due to disorder, and a dynamical transition separating a region with spatial chaos from a region with localized stable cooperative clusters emerges. Then, Wang et al. [48,49] again noticed this fact and studied the evolution of cooperation on sparse lattices, and found that the percolation threshold determines the optimal population density in spatial evolutionary games. But they only used a lattice with von-Neumann neighborhood (4 nearest neighbors). In addition, Ref. [50] also discusses the effect of group size on the cooperation behaviors among agents, but it is only based on the spatial public goods game and the influence of group size on the evolution of cooperation deserves to be further explored. In this paper, we combine our works on increasing neighborhood [51–53] with sparse lattices to further investigate the role of population density and increasing neighborhood in the evolution of cooperation on diluted lattices.

The rest of this paper is organized as follows. In Section 2, the game model with increasing neighborhood on diluted lattices is briefly introduced. Extensive numerical simulations are performed in Section 3, and the role of population density and increasing neighborhood is discussed in detail. Finally, the concluding remarks are given in Section 4.

## 2. Game model with increasing neighborhood on diluted lattices

In this work, we assume that each site on a square lattice can at most contain one player, that is, the actual number of players $N_{act}$ may be less than $N = L^2$ and some sites are empty during the process of evolution of cooperation. Initially, $N_{act}$ ($< N$) players are randomly distributed on the square lattice, and the population density or the effective fraction of players can be defined as follows: $\rho = \frac{N_{act}}{N} = \frac{N_{act}}{L^2}$, and the remaining sites $(1 - \rho)$ are vacant. Then we will consider the pairwise game model on diluted square lattices, such as the prisoner's dilemma game, the snowdrift game and the stag-hunt game, which depends on the ranking order in the payoff matrix. For simplicity, we try to reduce the complexity of the game model and employ a payoff matrix with only one parameter in all these three game models. For example, we choose the weak PDG model introduced by Nowak and May [22], that is, $T = b > 1, R = 1$ and $P = S = 0$, to investigate the cooperative behaviors of the PDG which almost captures all behaviors of a strict PDG although this model is only the boundary game between the strict PDG and SDG described in Ref. [54]; For the snowdrift game, we take $T = 1 + r, R = 1, S = 1 - r$ and $P = 0$ in which $0 < r < 1$ is the only parameter indicating the cost-to-benefit of cooperation. Meanwhile for the stag-hunt game, we can also use the normalized form: $T = r, S = -r, R = 1$ and $P = 0$ where $r$ has the same implication and value ranges as the SDG. In addition, each player can only adopt one of two possible strategies: cooperation ($s_i = C = 1$) or defection ($s_i = D = 0$) in these three games.

After the initialization, we will perform a Monte Carlo simulation (MCS) according to the following elementary step. First, a randomly selected agent $i$ calculates its payoff $p_i$ by playing the game with its possible $k$ neighbors, in which $k$ can be set



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