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Nonlinear-map model for the control of an airplane schedule

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HIGHLIGHTS

- We presented the nonlinear-map model for the airplane schedule.
- We studied the control and dynamics of the air transportation.
- We clarified the dependence of the airplane schedule on good quantity and control method.

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ABSTRACT

We study the dynamics and control of an airplane in air transportation between two airports. The dynamic models are presented for the airplane schedule. The dynamics of an airplane is described by the piecewise map and delayed map models. The characteristics of the nonlinear maps are studied analytically and numerically. The airplane displays the complex motion in the unstable region. The motion of the airplane depends on the quantity of goods, the control method, and the delay. It is shown that the control delay has an important effect on the motion.

1. Introduction

Recently, transportation systems have attracted much attention among physicists. Though traffic systems include many factors, physicists have proposed simplified traffic models including a few factors at most to clarify the cause and effect [1–7]. The physical traffic theory is an example of a highly quantitative description for a living system despite the complexity of traffic [8–17]. The concepts and techniques of physics are being applied to the traffic systems. The traffic models have been extended to take into account the traffic interruption and forecast effect by Tang et al. [18–21]. The vehicular traffic model has been extended to bus transportation. The traffic flow with bus stops has been studied by Tang et al. [22,23].

The traffic flow with many vehicles is a self-driven many-particle system. The jamming transitions and complex behaviors occur due to the many-body effect. However, the complex behavior appears due to the nonlinearity even in the traffic system with a single vehicle or a few vehicles. For example, the complex motion of buses is induced by the nonlinear interactions between buses and passengers in the shuttle bus transportation. The bus schedule is closely related to the nonlinear dynamics [24,25]. Also, elevator traffic has been investigated from the point of view of nonlinear dynamics [26–32].

Air transportation is similar to the shuttle bus or elevator traffic. However, air transportation has not been studied from the point of view of nonlinear dynamics. There are few dynamic models to predict the dynamic behavior of the airplane controlled by the waiting time. The airplane schedule is closely related to the dynamic behavior. It is important and necessary to know the dynamic behavior of the airplane transportation system. The airplane transportation system will be described by the nonlinear map model. The nonlinear map model will also be interesting from the point of view of nonlinear dynamics and chaos [33,34].

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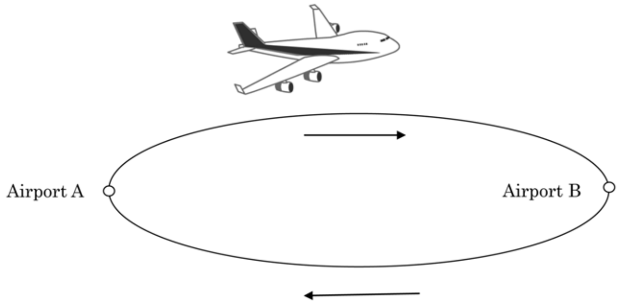

Fig. 1. Schematic illustration of air transportation. The airplane shuttles repeatedly between two airports A and B. Goods are loaded at airports A and B. Goods loaded at airport A (B) are unloaded at airport B (A).

In this paper, we study the dynamic behavior of the airplane transportation system controlled by waiting time synchronizing the quantity of goods (or passengers). We propose the nonlinear map model describing the airplane transportation system. We investigate the complex motion of the airplane. We show the dependence of the dynamic motion on the quantity of goods, the control method, and the delay.

2. Nonlinear map models

We consider the service of an airplane shuttling repeatedly between two airports A and B. We present the dynamic model which mimics the air transportation system. Fig. 1 shows the schematic illustration of air transportation. Passengers board the airplane at airport A and goods are loaded on the airplane at the same time. Then, the airplane moves from airport A to airport B. When the airplane arrives at airport B, all currently riding passengers leave the airplane. At the same time, all goods loaded at airport A are unloaded. After the airplane is empty, passengers waiting airport B board the airplane. At the same time, goods at airport B are loaded on the airplane. Then, the airplane moves from airport B to airport A. When the airplane arrives at airport A, all currently riding passengers leave the airplane and at the same time, all goods loaded at airport B are unloaded. After the airplane is empty, it stops at airport A in order to adjust the time schedule. Also, the airplane stops at airport A for a constant time for maintenance. If the operator cannot adjust the stopping time at airport A successfully, maintenance is omitted. Thus, the operator controls the time schedule for the air transportation system.

Furthermore, we assume that the time it takes to load the goods on the airplane is longer than the time it takes to board the passengers.

We describe the dynamic model of the airplane system in terms of the nonlinear map. First, we present the model for the simple case in which goods are loaded on the airplane only at airport A and there are no goods at airport B. We define the quantity of goods as $P_A(n)$ when the airplane arrives at airport A and trip n. One loads goods $P_A(n)$ on the airplane at airport A and trip n. When the airplane arrives at airport B, all goods $P_A(n)$ are unloaded from the airplane. We assume that the time it takes to load goods on the airplane is proportional to the quantity of goods. Then, the amount of time uploading all goods onto the airplane is given

$$\gamma P_A(n),$$

where γ is the time it takes to load one piece of goods onto the airplane. Similarly, the amount of time unloading all goods from the airplane is given by

$$\beta P_A(n),$$

where β is the time it takes to unload one piece of goods from the airplane.

If the boarding time of passengers is longer than that of goods, the boarding time is the number of passengers. In this model, the passenger's individual properties are not taken into account for simplicity. It is important to account for the individual properties of passengers. Tang et al. have studied the effect of the passenger's individual properties on the traffic flow [35,36]. Also, Tang et al. have proposed the aircraft boarding model [37,38]. In future work, it will be necessary to consider the detailed properties of passengers.

The moving time of the airplane is $2L/V$ where L is the distance between two airports and V is the mean speed of airplane. We define the stopping time at airport A and trip n for the maintenance and time adjustment as $S_A(n)$. The tour time equals the sum of these periods. Then, the arrival time $t(n+1)$ of the airplane at trip $n+1$ is given by

$$t_A(n+1) = t_A(n) + (\gamma+\beta)P_A(n) + \frac{2L}{V} + S_A(n). \tag{1}$$

The quantity $P_A(n)$ of goods at airport A and trip n is proportional to the tour time $\Delta t_A(n)$ $(=t_A(n) - t_A(n-1))$. The tour time is longer, the quantity of goods increases. The quantity is given by

$$P_A(n) = \mu \Delta t_A(n), \tag{2}$$

where μ is the rate of goods arriving at airport A.