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A thermodynamic counterpart of the Axelrod model of social influence: The one-dimensional case

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HIGHLIGHTS

- We report a thermodynamic version of the Axelrod model.
- We explicitly demonstrate the nonequilibrium nature of the original model.
- A unifying proposal is made to compare exponents across discrete 1D models.
- The presence of mass media carries the 1D system only to nonuniform regime.
- The 1D original model in N infinito remains in disordered state for any finite noise.

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ABSTRACT

We propose a thermodynamic version of the Axelrod model of social influence. In one-dimensional (1D) lattices, the thermodynamic model becomes a coupled Potts model with a bonding interaction that increases with the site matching traits. We analytically calculate thermodynamic and critical properties for a 1D system and show that an order–disorder phase transition only occurs at $T = 0$ independent of the number of cultural traits q and features F. The 1D thermodynamic Axelrod model belongs to the same universality class of the Ising and Potts models, notwithstanding the increase of the internal dimension of the local degree of freedom and the state-dependent bonding interaction. We suggest a unifying proposal to compare exponents across different discrete 1D models. The comparison with our Hamiltonian description reveals that in the thermodynamic limit the original out-of-equilibrium 1D Axelrod model with noise behaves like an ordinary thermodynamic 1D interacting particle system.

1. Introduction

The Axelrod model [1] was proposed originally to study dissemination of cultures among interacting individuals or agents. Although the model is too simple to simulate social dynamics, the mechanisms used in the model have been recognized by social scientists as a global self-reinforcing social dynamic [2]. It is a fact that the more culturally similar the people, the greater the chance of interaction between them, and that interaction increases their similarity [3]. These are the premises of the model.

More explicitly, the Axelrod model considers that an agent located at the ith site of a lattice is defined by a set of F cultural features (e.g., religion, sports, politics, etc.) represented by a vector $\sigma_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iF})$. Each feature σ_{ik} can take integer

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values in the interval $[1, q]$, where q defines the cultural traits allowed per feature and measures the cultural variability in the system. There are q^F possible cultural states. The model's dynamics is as follows: (1) Choose randomly two nearest neighbor agents i and j, then (2) calculate the number of shared features between the agents $\ell_{ij} = \sum_k^F \delta_{\sigma_{ik},\sigma_{jk}}$. If $0 < \ell_{ij} < F$, then (3) pick up randomly a feature k such that $\sigma_{ik} \neq \sigma_{jk}$ and with probability ℓ_{ij}/F set $\sigma_{ik} = \sigma_{jk}$. These time steps are iterated and the dynamics stops when a frozen state is reached; i.e., either $\ell_{ij} = 0$ or $\ell_{ij} = F, \forall i, j$. A cluster is a set of connected agents with the same state. Monocultural or ordered phases are composed of a cluster of the size of the system where $\ell_{ij} = F, \forall i, j$. Multicultural or disordered phases consist of two or more clusters.

One of the main features of this model is a change of behavior at a value q_c from a monocultural state, where all agents share the same cultural features, to a multicultural state, where individuals mostly have their own features [4]. This change can be characterized by an order parameter ϕ that is usually defined as the average size of the largest cultural cluster $C_{\max}$ normalized by the total number of agents N in the system; $\phi = C_{\max}/N$. In the monocultural (ordered) state $\phi \to 1$ and in the multicultural (disordered) state $\phi \to 0$.

The insertion of additional ingredients in the model, like an external field or mass media, yields interesting nontrivial consequences in the system [5–7]. The main limitation of the model seems to be that the system always converges to absorbent states, a situation that clearly does not occur in society. Some variants of the model relax this tendency by introducing noise into the system [8–10]. If the noise rate is small, the system reaches only monocultural states. However, if the noise rate is above a size-dependent critical value, a polarized state is sustained [8–10]. Klemm et al. [8,9] associated the monocultural (multicultural) states with stable (unstable) equilibria.

Until now, in the sociophysics field the global dynamics of social systems have been usually studied by postulating a series of rules that at the end lead to out-of-equilibrium behaviors, such as absorbent states. This approach often uses statistical mechanics concepts – temperature, critical phase transition, applied magnetic field, among others – without formal definitions. Langevin-type approaches have been proposed to study the collective phenomena of the social systems in terms of their microscopic constituents and their interactions [11]. Little attention has been paid to this approach in which the system can be modeled in a Hamiltonian formulation, whereupon equilibrium and nonequilibrium behaviors can be explored [12,13]. Such a Hamiltonian description also allows for the understanding of the meaning of social variables in the context of statistical mechanics.

Here, we develop a Hamiltonian version of the Axelrod model of social influence. Our Hamiltonian captures the local interactions of the original model. With the aim of finding a possible thermodynamic role of the parameters F, q, and q_c, our model, henceforth called *thermodynamic Axelrod*, uses the number of shared features ℓ_{ij} of the Axelrod model to construct a new Hamiltonian distinguishable from the 1D F-parallel-layer Potts models in that the interaction strength between agents increases with ℓ_{ij}. This feature of the interaction precipitates ordering preempting fluctuations. In the thermodynamic Axelrod model F is related to the coupling energy of the system and q has the same meaning as in the Potts model.

Although it is usually argued that the Axelrod model is an out-of-equilibrium model, this fact, taken as obvious, has never been demonstrated in the literature. In Section 2 we demonstrate that the standard Axelrod model does not satisfy the detailed balance condition. In Section 3 we analytically calculate the main thermodynamic functions for our model. For the critical behavior analysis (Section 4.2) we make an unifying proposal to compare exponents across different 1D discrete models, since current definitions depend on model details. In Section 5 we state the consequences of our study over the transitions driven by noise in the original Axelrod model. We discuss the implications of a thermodynamic society in Section 6 and in Section 7 we present our conclusions.

2. Axelrod model: out of equilibrium

Before we introduce the thermodynamic version of the Axelrod model, here we demonstrate that the original version does not satisfy equilibrium conditions by showing that detailed balance is violated.

Let a link between two sites i and j be of type n if they share n components ($\ell_{ij} = n$), P_n be the probability that the system is in a state with links of type n, and W_{nm} be the transition probability per unit time from a state with type-n to one with type-m links. W_{nm} being time independent. Since in the dynamics of the Axelrod model a feature k is changed with probability ℓ_{ij}/F to make two sites have one more component in common ($\sigma_{ik} = \sigma_{jk}$), we have

$$W_{nm} = \begin{cases} \dfrac{n}{F} & \text{for } m = n + 1, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The *detailed balance* relation implies that Ref. [14]

$$W_{nm} P_m = W_{mn} P_n \quad \forall\, n, m. \tag{2}$$

In the Axelrod model Eq. (2) cannot be satisfied since one side or the other is always zero according to Eq. (1). This feature of the model introduces some very strong constraints into both the evolution and the time-independent states of interactions that emphasize nonequilibrium; i.e. (i) completely different individuals do not interact, (ii) individuals conform once they have modified their cultural profile, and (iii) individuals that are alike, that interact, increase their similarity at interaction. These rules yield absorbing states, the most salient nonequlibrium feature of the Axelrod model.