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Studies on controllability of directed networks with extremal optimization

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HIGHLIGHTS

- General optimization problem statement of "control-over-network" has been proposed.
- The minimum control nodes problem for weighted–directed networks is studied.
- The minimum number of control nodes is related to the network's degree distribution.
- The evolution of the network topology is captured.

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ABSTRACT

Almost all natural, social and man-made-engineered systems can be represented by a complex network to describe their dynamic behaviors. To make a real-world complex network controllable with its desired topology, the study on network controllability has been one of the most critical and attractive subjects for both network and control communities. In this paper, based on a given directed–weighted network with both state and control nodes, a novel optimization tool with extremal dynamics to generate an optimal network topology with minimum control nodes and complete controllability under Kalman's rank condition has been developed. The experimental results on a number of popular benchmark networks show the proposed tool is effective to identify the minimum control nodes which are sufficient to guide the whole network's dynamics and provide the evolution of network topology during the optimization process. We also find the conclusion: "the sparse networks need more control nodes than the dense, and the homogeneous networks need fewer control nodes compared to the heterogeneous" (Liu et al., 2011 [18]), is also applicable to network complete controllability. These findings help us to understand the network dynamics and make a real-world network under the desired control. Moreover, compared with the relevant research results on structural controllability with minimum driver nodes, the proposed solution methodology may also be applied to other constrained network optimization problems beyond complete controllability with minimum control nodes.

1. Introduction

Networks give a natural representation of the structure of many complex systems that permeate many aspects of real-world physical, social, biological and computer network systems [1–4]. In general, a network is a collection of nodes, joined together in pairs by edges which depict some sort of relationship, e.g., Boolean, numerical, fuzzy, etc. In the Internet, nodes are routers which conduct the direction of information flow and edges are optical fibers which link routers physically, and

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in social networks, nodes are people and edges represent any type of social interactions including friendship, collaboration, affiliation or others. Over the past decade there has been an explosive growth of interest in both empirical studies of networks [5] and development of mathematical and computational tools for extracting insights from real-world network cases, tackling issues like modeling, topological property description, community detection, link prediction, synchronization and epidemic spreading analysis [6–17].

In the past decade, control over complex networks has been one of the attractive research areas for both network and control communities and many interesting and promising research results have been published [18–28] with the focus on the relationship between the network topology and its control dynamics. The dynamics of a network can be represented by a canonical time-invariant linear system $\dot{\mathbf{X}}(t) = A\mathbf{X}(t) + B\mathbf{u}(t)$ [18,29,30], where $\mathbf{X} \in R^N$ is the state vector of the network, $\mathbf{A} \in R^{NXN}$ is the state matrix describing the linking weights between state nodes, $\mathbf{B} \in R^{NXP}$ is the input matrix depicting the coupling strength between state nodes and control nodes and $\mathbf{u} \in R^P$ is the input vector of the network, here assuming the network is of N state nodes and P control nodes. The detailed explanation of these notations can be found in the following section. A network is said to be completely controllable if, with a suitable choice of inputs, it can be guided from any initial state $\mathbf{X}_o$ to any desired state $\mathbf{X}_f$ within finite time, which can be numerically judged by the Kalman controllability rank condition, namely rank$(\mathbf{C} = (\mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B}, \dots, \mathbf{A}^{N-1}\mathbf{B})) = N$, where $\mathbf{C}$ is the controllability matrix. In certain cases, the precise value of rank$(\mathbf{C})$ is difficult to determine because the elements of $\mathbf{A}$ and $\mathbf{B}$ are often not known accurately except the zeros that mark the absence of connections between nodes [18]. Hence $\mathbf{A}$ and $\mathbf{B}$ are often considered to be structured matrices, i.e. their elements are either fixed zeros or independent free parameters. Obviously, the rank$(\mathbf{C})$ varies as a function of the free parameters of $\mathbf{A}$ and $\mathbf{B}$. If there exists one set of values of these free parameters that can make rank$(\mathbf{C}) = N$, then the network is structurally controllable [31]. It deserves to emphasize that a network said to be structurally controllable will not be completely controllable under some pathological sets of values of free elements of $\mathbf{A}$ and $\mathbf{B}$, whose rank$(\mathbf{C}) < N$.

Among other publications on network control, Liu et al. [18] developed a tool for structural controllability of complex networks with minimum number of driver nodes, N_D. The simulation results with various real-world and computer-generated networks show that N_D is determined mainly by the network's degree distribution and the sparse inhomogeneous networks are the most difficult to control. In addition, Pósfai et al. [19] found the effects of the network's underlying degree correlation on N_D, Wang et al. [20] optimized N_D by making the minimum structural perturbations to the network, and Nepusz and Vicsek [21] explored the network controllability by examining edge dynamics in systems for which a state variable is associated to each edge. Two node significance indexes, control centrality and control range, were proposed to characterize the ability and responsibility of one single node in controlling the whole network, respectively [22,23], and the robustness of network controllability was also addressed [26], revealing that degree-based attacks are more efficient than random attacks on network structural controllability.

It is realized that the research activities as noted above mainly focus on the structural controllability [31] for directed networks only with state nodes being linked mainly by Boolean edges. These studies assume the weights of connections of the networks are precisely unknown and the networks taken into consideration have no control configuration, i.e. they are only consisting of state nodes. The main goal is to design the proper network configuration that keeps its structural controllability. For example, in Ref. [18], Liu et al. take advantage of the graph-theoretical approach to figure out the minimum number of state nodes of the given networks, for which linking one single control node can guarantee the whole network's structural controllability.

However, most real-world complex systems, e.g., urban transportation system, natural gas pipeline network, power grids, etc., physically consist of both state nodes and control nodes. In addition, the linking strength between state nodes and/or between control nodes and state nodes could be reasonably determined by principle knowledge, big-data analysis and/or human professional experience. For example, in the natural gas pipeline network, the user terminals and the compressor stations are represented by state nodes and control nodes respectively, shown in Fig. 1. This kind of directed–weighted networks is popular in various physical, social, economic, biological and man-made-engineered systems. Therefore, it is worthwhile to investigate the proper control policy over them under the given control configuration with the desired criteria, such as the minimum control nodes and/or minimum control energy, etc., to generate the optimal network topology, while making the whole network completely controllable, not only structurally controllable.

In this paper, for a given directed–weighted network consisting of both "state nodes" and "control nodes", we propose the general problem formulation of 'control-over-network' with its desired criteria to generate an optimal network topology subject to the constraints. As an initiative research activity, the minimum control nodes (MCN in short) for complete controllability is defined as a criterion subject to a given initial network topology.

Mathematically, the MCN problem defined here can be mapped to a constrained combinatorial optimization problem with binary decision variables which determine the selection of control nodes to form a desired network topology. Assuming a directed–weighted network with N state nodes and P control nodes, a brute-force search of the control node configurations will require us to test $2^P - 1$ distinct combinations using the canonical Kalman's controllability rank criterion, which is a computationally prohibitive task for large networks. Here we develop a novel heuristic tool under the framework of extremal optimization (EO), which was inspired from the far-from-equilibrium dynamics of self-organized criticality (SOC) [33–35] and has been successfully applied to a variety of hard combinatorial optimization problems [36–40] such as the traveling salesman problem, spin glass, and graph partitioning. Simulation results on the different types of networks with various degree distributions demonstrate the effectiveness of this method, and in addition unveil some interesting relationships between the network's underlying structural properties and the MCN.