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Analysis of energy dissipation in traffic flow with a variable slope

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HIGHLIGHTS

- We derived an analytical energy dissipation model for traffic flow.
- We found the relationship between the total energy consumption and the slope.
- We found that energy dissipation rate depends on traffic density and road length.

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ABSTRACT

We derive the motion energy dissipation model to investigate the relation between the additional energy loss of vehicles and the slope of a gradient. Simulations are carried out to check the validity of the dissipation model. Analysis of the results shows that the total energy consumption is inversely proportional to the slope in an uphill situation and the opposite conclusion can be drawn in a downhill situation. The energy dissipation rate depends on the density of traffic and the road length in two situations in a rule. It is found that the simulation result is in good agreement with real traffic.

1. Introduction

Traffic problems become more and more serious with an increase in the number of private cars number in modern city. Congestion, energy dissipation, environmental pollution and accidents are brought about due to heavy traffic. In particular, energy dissipation caused by congestion has become an attractive issue investigated by many physicists and scholars in recent years [1–5]. Nakayama et al. [1] examined the validity of the traffic flow model by estimating the additional energy loss by using the following quantity: $E = \sum_{\text{vehicles}} \sum_{\text{waves}} \frac{1}{2}(v_{\max}^2 - v_{\min}^2)$. Lower energy consumption means a more effective traffic flow model in suppressing the congestion. The above equation is related to the energy dissipation of the whole traffic flow. However, Shi and Xue [2] thought that it is more important to consider the energy consumption of each vehicle in understanding traffic jamming and congestion dispersion. From this point of view they systematically analyzed the relation between the stability and energy consumption in several typical optimal velocity models. Zhang et al. [3] numerically investigated the energy dissipation per unit time per car in the Nagel–Schreckenberg (NS) model. They found that energy dissipation depends on both the vehicle density and the initial condition in the deterministic NS model, while in the non-deterministic case the initial configuration has no effect on energy dissipation. Tian et al. [4] and Wen et al. [5] respectively investigated the energy dissipation of mixed traffic flow based on cellular automaton NS and FI (Fukui-Ishibashi) models. They found that energy dissipation depends on the maximum velocity, vehicle length, ratio of mixed vehicles, delay probability, and so on. Moreover, it is pointed out that FI model is different from the NS model in describing the loss of motion energy.

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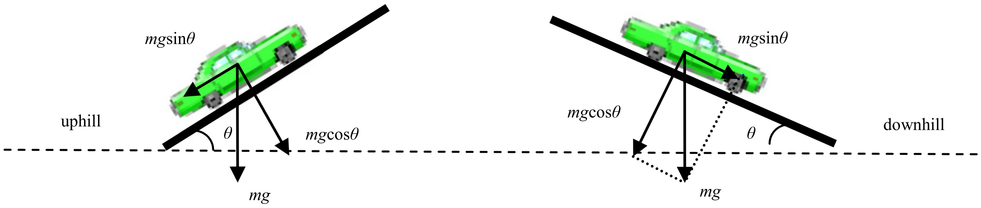

Fig. 1. Illustration of a vehicle running on gradients: uphill and downhill.

The aforementioned literature mainly focused on the energy dissipation of vehicles moving on an even road. There is no doubt that the energy dissipation of vehicles on a road with gradient is different from that of vehicles on an even road. On roads with different gradients the energy dissipation of vehicles should vary with the slope in an unstable condition. In the following we will discuss the relation between energy dissipation and the slope of the road.

The remainder of the paper is organized as followed. In Section 2 the energy dissipation model is derived. In Section 3 simulations are carried out in four situations and analysis of the results is also given. The last part is the summary.

2. Energy dissipation model

In order to derive the relation between the energy dissipation rate and the slope of the road section we will combine the energy dissipation formula with a car-following model with a slope effect to formulate the dissipated energy of a vehicle on a road with gradient.

From Ref. [2] the kinetic energy of a vehicle is up to its velocity v formulated as $(mv^2)/2$, where m is the mass of the vehicle. The loss of kinetic energy is defined as the energy dissipation caused by the energy that is lost if the vehicle decelerates. According to the conservation law of kinetic energy, in a steady state the increase in energy while accelerating equals the dissipated energy due to the deceleration. Neglecting rolling and air drag dissipation and other dissipation such as the energy to keep the motor running, we only take the motion energy loss caused by the deceleration.

The dissipated motion energy ΔE of the nth vehicle in an N-vehicle system from $t-1$ to t is defined as follows:

$$\Delta E_n(t) = \begin{cases} \frac{1}{2} m\left[v_n^2(t-1) - v_n^2(t)\right], & \text{if } v_n(t) < v_n(t-1), \\ 0, & \text{if } v_n(t) \geq v_n(t-1). \end{cases} \tag{1}$$

Thus, the motion energy dissipation rate is

$$E_m = \frac{1}{T}\frac{1}{N}\sum_{t=t_0+1}^{t_0+T}\sum_{n=1}^{N}\Delta E_n(t) \tag{2}$$

where N is the total number of vehicles in the system, and t_0 is the relaxation time, taken as a long enough time for the system to reach a steady state. T is the time period for calculating total energy dissipation.

The car-following model, as an analytical model that can be used to analyze the motion of vehicles in different conditions [6–21] including gradient situations [22–25], could be used to study the relation between the energy dissipation and the slope. Li et al. [22] investigated the phase transition on speed limit traffic with the slope based on Bando's optimal velocity model [7]. Komada et al. [23] investigated the effect of the gravitational force upon traffic flow on a slope with sags, uphill and downhill, and derived the fundamental diagram for the traffic flow in the sag. Lan et al. [24] investigated the effect of slopes with use of a cellular automaton model. Some new results have been derived from theoretical analysis and simulation. Zhu and Yu [25] analytically and numerically investigated the shock waves, soliton waves and kink waves which are described by the Burgers, KdV and modified KdV equations in different regions. Although all of the above aforementioned models can be used to analyze the energy dissipation of traffic flows on a road with gradient, in this paper we choose the model in Ref. [25] to investigate the relation between the energy consumption and the slope.

A vehicle moving on a single lane road with gradient is shown in Fig. 1 in two situations. The slope of the gradient is represented by θ, the gravitational acceleration is g and the mass of a vehicle is m. Then, a horizontal force $mg\sin\theta$ acts upon the vehicle at any time.

The motion equation of a vehicle on a single lane gradient was formulated as

$$\frac{dv_n(t)}{\mathrm{d}t} = \frac{1}{\tau}\left\{V\left(\Delta x_n, \theta\right) - v_n(t)\right\} \tag{3}$$