Physica A 392 (2013) 3344–3350

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Time-series analysis of foreign exchange rates using time-dependent pattern entropy

Ryuji Ishizaki [a,*], Masayoshi Inoue [b]

[a] Faculty of Integrated Human Studies and Social Sciences, Fukuoka Prefectural University, Tagawa 825-8585, Japan
[b] Professor Emeritus of Kagoshima University, Department of Physics, Kagoshima 890-0065, Japan

HIGHLIGHTS

- Time-dependent pattern entropy (T-DPE) is a new method of time-series analysis.
- The T-DPE was applied to the time series of daily variation in the dollar–yen rate.
- When T-DPE remains high, it indicates that there is confusion in the market.
- The T-DPE was able to automatically extract unstable periods in the dollar–yen rate.

ARTICLE INFO

Article history:
Received 27 July 2012
Received in revised form 12 January 2013
Available online 15 April 2013

Keywords:
Time-dependent pattern entropy
Financial time series
Exchange rate
Symbolic dynamics

ABSTRACT

Time-dependent pattern entropy is a method that reduces variations to binary symbolic dynamics and considers the pattern of symbols in a sliding temporal window. We use this method to analyze the instability of daily variations in foreign exchange rates, in particular, the dollar–yen rate. The time-dependent pattern entropy of the dollar–yen rate was found to be high in the following periods: before and after the turning points of the yen from strong to weak or from weak to strong, and the period after the Lehman shock.

1. Introduction

Complex systems have recently received considerable attention, and the foreign exchange market can be regarded as a typical example of such a system [1,2]. Time series of foreign exchange rates are typically nonstationary and exhibit complicated variations. The construction of a new method for extracting useful information from such nonstationary time series would fill an important need for analyzing a wide range of complex systems.

In a recent paper, we applied time-dependent pattern entropy (T-DPE) to electroencephalograms (EEG) of rats to measure their level of consciousness [3]. This method has been found to be useful for measuring the level of consciousness quantitatively, and it could lead to the development of new methods of sleep analysis. It has also been applied to multichannel human EEGs, and levels of consciousness have been estimated by using entropy [4]. A different definition of time-dependent entropy was presented by Tong et al., who studied the EEGs of rats with brain injuries caused by cardiac arrest [5,6]. They used the Tsallis entropy form [7,8], and the entropy was calculated by using the probability of the EEG amplitude distribution for a sliding temporal window.

Kashima and one of the authors used T-DPE to investigate the dynamics of a chaotic neural network [9]. They showed that entropy characterizes the temporal behavior of self-organization in a neural network that is solving a traveling salesman problem.

* Corresponding author. Tel.: +81 947 42 2118.
E-mail address: ishizaki@fukuoka-pu.ac.jp (R. Ishizaki).

0378-4371/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physa.2013.03.041

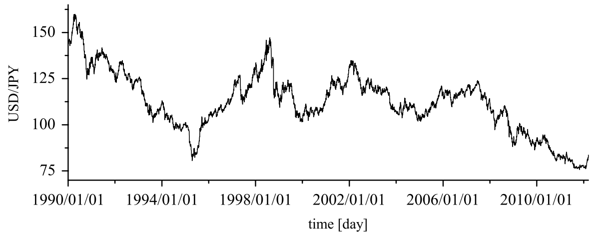

Fig. 1. Time-series of the daily USD/JPY bid rates $\{x_n\}$ for the period 1 Jan 1990–16 Mar 2012.

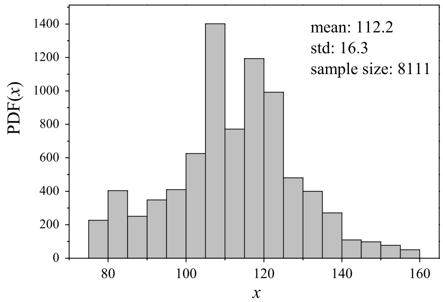

Fig. 2. Histogram of daily bid rates $\{x_n\}$ of USD/JPY, using data for the period 1 Jan 1990–16 Mar 2012.

In the present paper, we propose the use of T-DPE as a simple method to automatically quantify the instability in the time series of foreign exchange rates. To analyze the time series of foreign exchange rates as a physical phenomenon, we first transform it into a binary symbolic time series and then use an embedding method to calculate the T-DPE. This method resembles the technique of embedding chaotic time-series for analysis [10]. Using this approach, T-DPE can measure the complexity of the patterns of short-term time series of foreign exchange rates.

In Section 2, we consider the statistical properties of the dollar–yen (USD/JPY) exchange rate. In Section 3, we show how to obtain the T-DPE from the time series of foreign exchange rates, and in Section 4, we obtain it for the daily variation in the USD/JPY exchange rate. The final section is devoted to a short summary of our results.

2. Statistical properties of USD/JPY

In this section, we discuss the long-time average statistical properties of the US dollar–yen (USD/JPY) exchange rate, such as the time-correlation function and the empirical probability density function. We used the daily USD/JPY bid rates $\{x_n\}$ provided by the Oanda Corporation [11]. The sample period is 1 Jan 1990 through 16 Mar 2012. This corresponds to a sample size of 8, 111 days (Fig. 1). Characterizing the statistical properties of the daily data $\{x_n\}$ of the USD/JPY exchange rate by using the first and second moments for $\{x_n\}$ (see the histogram of $\{x_n\}$ in Fig. 2) is difficult. This is because the daily data $\{x_n\}$ of the USD/JPY exchange rate is a nonstationary time series. Therefore, we investigated the successive differences $\{\ln x_{n+1} - \ln x_n\}$, which is a common method for removing a trend in order to characterize the statistical properties of a nonstationary time series. The reason for this choice is as follows. Unit root tests can be used to determine whether trending data should be first differenced to render the data stationary. The Phillips–Perron tests consider the null hypothesis that a time series has a unit root. The p-value of the Phillips–Perron test is 0.5406 for the time series $\{\ln x_n\}$. Since the presence of a unit root was not rejected, we applied the difference operator to the time series $\{\ln x_n\}$ and took the successive differences $\{\ln x_{n+1} - \ln x_n\}$ in order to remove trends. Thus, we investigated

$$r_n \equiv \ln x_{n+1} - \ln x_n = \ln \frac{x_{n+1}}{x_n}. \tag{1}$$

If x is a function of time t, the time series of logarithmic returns of $x(t)$ over a short time scale τ is

$$\ln \frac{x(t+\tau)}{x(t)} = \ln \left[1 + \frac{\Delta x}{x(t)}\right] \simeq \frac{\Delta x}{x(t)} \quad \text{for } \Delta x \ll x. \tag{2}$$

Here r_n, which is not affected by changes in the scale, is the rate of increase in x_n per unit time.

The time-correlation function of r_n is given by

$$C(n) \equiv \langle (r_n - \langle r_n \rangle)(r_0 - \langle r_0 \rangle) \rangle, \tag{3}$$