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Modified circle map model for complex motion induced by a change of shuttle buses

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HIGHLIGHTS

- We studied the bus dynamics induced by a change of shuttle buses.
- We presented the modified circle map model for the bus transportation system.
- We clarified that the shuttle bus shows such complex behavior as periodic, quasi-periodic, and chaotic motions.

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ABSTRACT

We investigate the dynamic behavior of shuttle buses when passengers switch to another bus B on route B from bus A on route A. By switching from bus A to bus B, the outflow of passengers from route A (inflow of passengers into route B) changes to the periodic inflow of a square wave. The dynamics of the shuttle buses with the change is described by the modified circle map model. The bus schedule and control are closely related to the dynamics. The motion of shuttle buses depends on the inflow rate, its period, and moving time ratio. The shuttle bus displays such complex behavior as periodic, quasi-periodic, and chaotic motions.

1. Introduction

Recently, transportation problems have attracted much attention among physicists [1–5]. The transportation system displays interesting dynamic behaviors. The nonlinear discrete dynamics has been studied in the traffic flow, pedestrian flow, and bus-route problems [6–31]. The quasi-periodicity and chaos have been found as a typical signature of the complex behavior of transportation system [30–33]. The bus-route problems have been investigated by using the cellular automaton model and the car-following model. It has been found that the bunching (or clustering) of buses occurs by interacting between buses and passengers [17–20].

Real traffic is very complex. In the city traffic network, the OD problem is important for a route choice. In a real traffic network, the driver's property, forecast effect, road conditions, and bus stations have an important effect on the vehicular traffic. It is necessary to extend a simple traffic model to complex traffic. Tang et al. have studied network traffic flow using road traffic theory [34]. Tang et al. have proposed some models to explore the bus-following behavior and the effect of a bus station on traffic flow [35–37]. Furthermore, Tang et al. have developed some models with consideration of the forecast effect, road conditions, and a driver's individual properties [38–44].

The traffic flow with many vehicles is a self-driven many-particle system. The jamming transitions and complex behaviors occur due to the many-particles effect. However, the complex behavior appears due to the nonlinearity even in the

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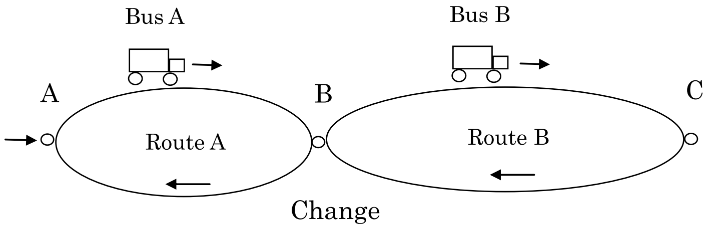

Fig. 1. Schematic illustration of two shuttle buses with a change. There are two routes of buses. Bus A repeatedly serves transfer station B from starting terminal (origin) A on route A. Another bus B repeatedly serves final terminal (destination) C from transfer station B on route B.

transportation system with a single bus or a few buses. The bus schedule is closely related to the nonlinear dynamics. It is important and necessary to make the bus schedule. The shuttle bus transportation has been studied for the bus schedule [25,26,30,31]. When the buses repeatedly shuttle between the starting terminal (origin) and the final terminal (destination), it is necessary to estimate the arrival time of buses accurately for a bus schedule. The bus dynamics has been described in terms of the nonlinear map. It has been shown that the shuttle bus transportation displays the complex motion by the interaction between buses and passengers. Also, it has been found that the deterministic chaos occurs in the shuttle-bus transportation by the speedup [25,26]. When the inflow rate of passengers into the bus varies with time periodically, the bus dynamics has been described by the circle map of sine wave [45]. Also, in the transportation of shuttle buses controlled by the capacity, the dynamics has been described by the nonlinear map model. It has been shown that the bus schedule is determined by the piecewise map [46].

In real bus traffic, buses run simultaneously on some routes. Generally, one will go to a destination through a transfer point by switching to another bus. The dynamic motion of buses will vary with a change of buses because buses interact with passengers. However, there are few dynamic models to predict the dynamic behavior of buses for the transportation with a transfer station for changing buses.

In this paper, we study the dynamic behavior of the shuttle buses when passengers change from route A to route B through a transfer stop. We describe the shuttle bus transportation with a change of buses in terms of the modified circle map of square wave. We show that the shuttle bus displays periodic, quasi-periodic, and chaotic motions. We clarify the dependence of the dynamic motion on the inflow rate, its period, and moving time ratio.

2. Model and nonlinear map

We consider the bus transportation when passengers go to the destination by changing from one bus to another bus. Fig. 1 shows the schematic illustration of two shuttle buses with a change. There are two routes for buses. Bus A serves repeatedly transfer station B from starting terminal (origin) A on route A. Another bus B serves repeatedly final terminal (destination) C from transfer station B on route B. Starting terminal A is the way to board the bus. Passengers come into the starting terminal at a constant rate, board the bus stopping at the origin, the bus moves toward transfer station B, and passengers get off the bus at transfer station B when bus A arrives at the transfer place. After all passengers get off bus A at transfer point B, bus A with no passengers moves again toward origin A. Bus A shuttles repeatedly between starting terminal A and transfer station B. The passengers at the transfer point wait for the next bus B on route B to go to final terminal (destination) C. When bus B arrives at transfer station B, passengers waiting at transfer point B board the bus and bus B moves toward final terminal C. When bus B arrives at final terminal, all passengers get off. Then, bus B with no passengers moves again toward transfer station B. Bus B shuttles repeatedly between transfer station B and final terminal C.

We describe the dynamic model for two shuttle buses with a change in terms of a nonlinear map. We assume that passengers come into the origin at constant inflow rate μ. Define the number of passengers boarding bus A at trip n by $B_A(n)$. The parameter γ is the time it takes one passenger to board the bus, so $\gamma B_A(n)$ is the amount of time needed to board all the passengers at the origin. The moving time of bus A is $2L_A/V_A$ where L_A is the length between origin A and transfer point B and V_A is the mean speed of bus A. The stopping time at transfer point B to leave off the passengers is $\beta B_A(n)$ where parameter β is the time it takes one passenger to leave the bus. The travel time of bus A equals the sum of these periods. Therefore, the arrival time $t_A(n+1)$ of bus A at the origin on trip $n+1$ is given by

$$t_A(n+1) = t_A(n) + \gamma B_A(n) + \frac{2L_A}{V_A} + \beta B_A(n). \tag{1}$$

Define $W_A(n)$ as the number of passengers waiting at the origin just before the bus arrives at the origin on trip n. New passengers arrive at the origin with rate μ_A. So $W_A(n)$ is the number of passengers that have arrived since the previous bus left the origin. This is expressed by

$$W_A(n) = \mu_A \{t_A(n) - t_A(n-1)\}. \tag{2}$$

If one assumes that the capacity of the bus is sufficiently large, the number of passengers boarding the bus is consistent with the number of passengers waiting at the origin

$$B_A(n) = W_A(n). \tag{3}$$