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Periodic orbit theory in fractal drums

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Abstract

The level statistics of pseudointegrable fractal drums is studied numerically using periodic orbit theory. We find that the spectral rigidity $\Delta_3(L)$, which is a measure for the correlations between the eigenvalues, decreases to quite small values (as compared to systems with only small boundary roughness), thereby approaching the behavior of chaotic systems. The periodic orbit results are in good agreement with direct calculations of $\Delta_3(L)$ from the eigenvalues.

Keywords: Quantum chaos; Billiards; Fractal drums; Pseudointegrable systems

1. Introduction

Many systems with irregular geometries exist in nature and their physical properties (e.g. their electronic and vibrational behavior) have been widely investigated in the past. One class of irregular systems are those that are formed by an ordered material, but possess an irregular shape of the boundary. A prominent example are the fractal drums [1] which are constructed by applying the so-called fractal generator (see Fig. 1(a)) ν times to a regular square or rectangle. Already for prefractal shapes as e.g. for $\nu = 1$–3, the eigenstates possess energy spectra and localization properties that are well distinct from systems with smooth boundaries.

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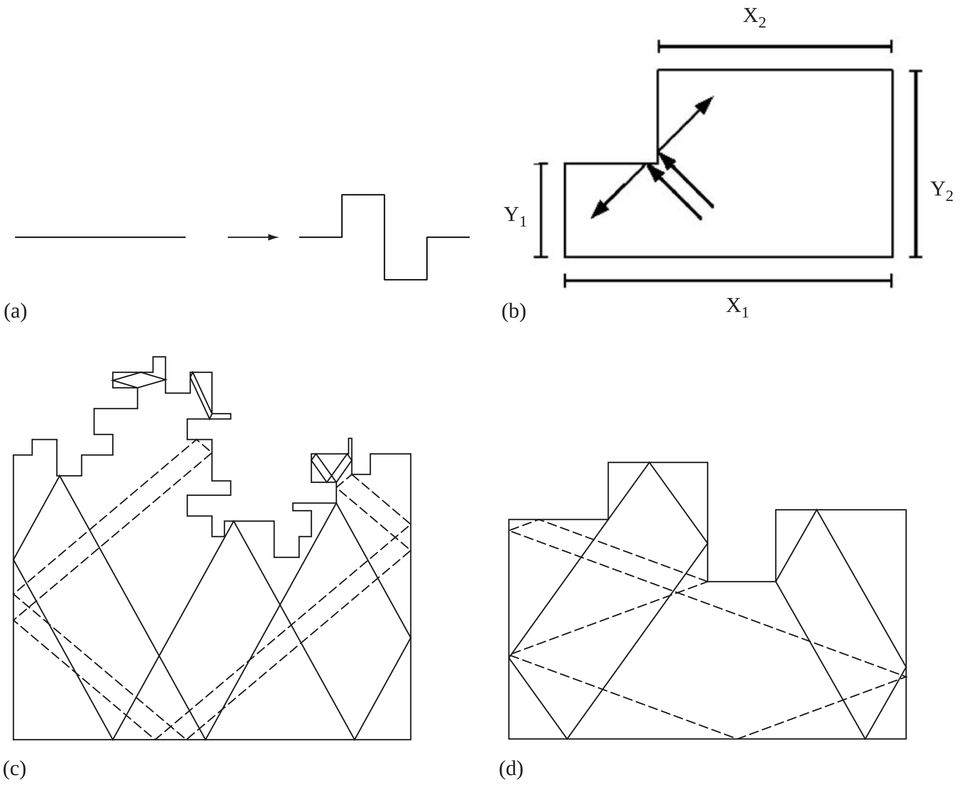

Fig. 1. Geometry and periodic orbits. (a) The fractal generator. (b) The L-shaped system of $g = 2$. (c) Fractal drum of $v = 2$ with the generator applied to the upper border and $g = 28$. (d) Fractal drum of $v = 1$ with the generator applied to the upper border and $g = 4$. The segments of the drums are slightly deformed in order to avoid degeneracies of the orbit lengths. Some selected periodic orbits are shown for each geometry and the beam-splitting property of the salient corners and the different segments X_1, X_2, Y_1 and Y_2 are demonstrated in (b).

This has been demonstrated by numerical simulations [2–4] as well as by experiments on liquid crystal films [4] and acoustic cavities [5]. In this paper, we address the question of how the specific geometry of the systems influences the behavior of the energy spectra within periodic-orbit theory, which provides a link between the classical and the quantum mechanical behavior of systems with the same shape.

Two universality classes exist with different classical dynamics, the chaotic and the regular systems. Fractal drums belong to the intermediate class of pseudointegrable systems, which are polygons with only rational angles $n_i\pi/m_i$, with $n_i, m_i \in \mathbb{N}$ and at least one $n_i > 1$ (see, e.g. [6–13] and references therein). They are characterized by their genus number g, which is equal to 1 for integrable systems and ∞ for chaotic systems. For pseudointegrable systems, $1 < g < \infty$. In the specific case of systems with only right angles, $g = 1 + N_{3\pi/2}$, where $N_{3\pi/2}$ is the number of angles $3\pi/2$.