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Fixing the fixed-point system—Applying Dynamic Renormalization Group to systems with long-range interactions

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ABSTRACT

In this paper, a mode of using the Dynamic Renormalization Group (DRG) method is suggested in order to cope with inconsistent results obtained when applying it to a continuous family of one-dimensional nonlocal models. The key observation is that the correct fixed-point dynamical system has to be identified during the analysis in order to account for all the relevant terms that are generated under renormalization. This is well established for static problems, however poorly implemented in dynamical ones. An application of this approach to a nonlocal extension of the Kardar–Parisi–Zhang equation resolves certain problems in one-dimension. Namely, obviously problematic predictions are eliminated and the existing exact analytic results are recovered.

1. Introduction

Fluctuating surfaces appear in a wide variety of physical situations and have been of great interest in the past two decades [1–3]. These and other systems far from thermal equilibrium pose a major challenge in contemporary statistical physics. Behavior out-of-equilibrium is far richer than at equilibrium, and many intriguing scaling phenomena, such as self-organized criticality [4], or phase transitions between non-equilibrium stationary states [1], have been observed for long. However, despite the considerable achievements, the theoretical comprehension of non-equilibrium phenomena remains much poorer than our understanding of equilibrium phenomena.

The Renormalization Group (RG), proven useful to explain universality in equilibrium continuous phase transitions, has also allowed some progress in understanding systems out-of-equilibrium. Nevertheless, in many cases the information RG analysis offers in not complete and limited to a certain range of dimensions. A classical example is the Kardar–Parisi–Zhang (KPZ) equation [3] where the Dynamic Renormalization Group (DRG) approach agrees with the analytic exact result in one dimension [1] but unable to provide results for the strong coupling phase in higher dimension. This clearly indicates that internal problems exist in the DRG calculation for $d > 1$. Actually, a remarkable result of Wiese [5] shows that the shortcoming of DRG in the KPZ system is not an artifact of a low order calculation (so called "one loop" calculation), but rather intrinsic to the method and extends to all orders. This situation motivated the development of other methods to deal with the KPZ system such as a scaling approach [6], Self-Consistent Expansion (SCE) [7], Mode-Coupling [8] and others that were able to provide predictions for the exponents in more than one-dimension.

A decade ago, a family of nonlocal growth models has been introduced in Ref. [9], known as the Nonlocal KPZ (NKPZ) equation, to account for nonlocal interactions in a system of deposited colloids, giving rise to roughness larger than the one

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predicted by the classical KPZ case. The authors studied the white noise case that was later generalized to spatially correlated noise in Ref. [10]. To be more specific, the equation they studied was

$$\frac{\partial h\left(\vec{r},t\right)}{\partial t} = \nu\nabla^2 h + \frac{\lambda_\rho}{2}\int d^d r' \frac{\nabla h(\vec{r}+\vec{r}')\cdot\nabla h(\vec{r}-\vec{r}')}{\left|2\vec{r}'\right|^{d-\rho}} + \eta\left(\vec{r},t\right), \tag{1}$$

where $\eta\left(\vec{r},t\right)$ is a noise-term modeling the fluctuation of the rate of deposition, which has a zero mean and is characterized by its second moment

$$\left\langle \eta\left(\vec{r},t\right)\eta\left(\vec{r}',t\right)\right\rangle = 2D_0\left|\vec{r}-\vec{r}'\right|^{2\sigma-d}\delta\left(t-t'\right), \tag{2}$$

where d is the substrate dimension and D_0 specifies the noise amplitude. Note that in the limit $\rho \to 0$ the local KPZ equation is recovered. This model suggests that the growth at each point $\vec{r}$ gets contributions from pairs of gradients at points symmetrically located around $\vec{r}$ along the interface, namely $\nabla h(\vec{r}\pm\vec{r}')$, with a weight that is a decreasing function of the distance between them.

Both papers [9,10] have investigated this problem using the Dynamic Renormalization Group (DRG) approach, and have derived a complex phase diagram. Focusing on the strong coupling solution (in the KPZ sense [1,3]) both papers have found

$$z = 2 + \frac{(d-2-2\rho)\,(d-2-3\rho)}{(3+2^{-\rho})\,d-6-9\rho}, \tag{3}$$

where z is the dynamic exponent. The roughness exponent, α, characterizing the long distance spatial behavior, is obtained using the modified Galilean scaling relation $\alpha + z = 2 - \rho$. Unfortunately, the DRG result for the exponents, summarized in Eq. (3) above, was found to be inconsistent with an exact inequality in a certain range of the parameters and in all dimensions [11]. It is in place to comment here on the possible violations of the modified Galilean scaling relation, in view of recent criticisms of the relation between Galilean invariance and the scaling relation in the original KPZ system [12–14]. This may be especially relevant for discretized versions of Eq. (1) (see Ref. [14]) and less so in the continuum limit, which is the main focus of this paper.

Interestingly, another nonlocal extension of the KPZ equation has been studied in the literature [15,16], namely

$$\frac{\partial h\left(\vec{r},t\right)}{\partial t} = \nu\nabla^2 h + \frac{\lambda_\rho}{2}\int d^d r' \frac{\nabla h(\vec{r}')\cdot\nabla h(\vec{r}')}{\left|\vec{r}-\vec{r}'\right|^{d-\rho}} + \eta\left(\vec{r},t\right). \tag{4}$$

This model also recovers the standard KPZ equation in the limit $\rho \to 0$, but suggests that the growth at every point $\vec{r}$ comes from the contribution of the gradients at all the points on the interface with a relative weight that decreases with the distance to $\vec{r}$. This is different from Eq. (1) in that the nonlinearity contributes to growth via the local interaction with all the other points on the interface, and not just pairs of points symmetrically distributed around it.

It turns out that this model enjoys an exact result in 1D [15] predicting $z = (3-3\rho)/2$ when $\rho = 2\sigma$. It also happens that the same scaling relation $\alpha + z = 2 - \rho$ holds here, from which the roughness exponent α could be worked out. A more systematic study using the Self-Consistent Expansion [16] agrees with the exact result when applicable, and provides predictions for the exponents in other dimensions as well. On physical grounds this seems to be a simpler nonlocal extension of the KPZ nonlinearity than that of Eq. (1), and therefore worthwhile understanding when modeling systems with long-range interactions. This simplicity is reflected in the fact that the nonlinear term in Eq. (4) is more "relevant" (in the Renormalization Group sense) than the one in Eq. (1), as will be seen below. The interesting thing is that the scaling dimension of the linear and nonlinear terms does not coincide in this equation, and this hinders the direct application of the perturbative Renormalization Group analysis. A key observation made in the SCE analysis [16], and which will be helpful for the DRG analysis as well, is that super diffusive modes of relaxation are generated by the nonlinearity of Eq. (4), namely super diffusion modes. This suggests that a remedy should be sought going back the old Renormalization idea of identifying first the right fixed point dynamical system around which the expansion should be. The fixed point dynamical system is not necessarily of the same form as the original system, as is implicitly assumed by the standard DRG procedure. That terms not included in the original action can be generated under the renormalization group has been known from the very beginning of the renormalization group, and was taken into account in static problems [17–19]. However, this is often overlooked in dynamical problems.

In this paper, a modification of the standard DRG procedure that goes along those lines is suggested. This approach makes DRG more flexible, and succeeds in recovering the exact result for the case of the NKPZ Eq. (4). Not less important, this approach could be useful in implementing DRG in other situations where long-range interactions are present, such as those appearing in the context of hydrodynamic interactions in colloidal suspensions [20,21], nonequilibrium fluctuations of an interface under shear [22], wetting of an amorphous solid by a liquid [23,24] and in in-plane tensile crack propagation in a disordered medium [25,26]. The main motivation here is to make the first step towards extending the range of applicability of DRG in a field that suffers anyway from a lack of analytical tools, in order to allow further progress in systems out-of-equilibrium.