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Some dynamical properties of a classical dissipative bouncing ball model with two nonlinearities

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ABSTRACT

Some dynamical properties for a bouncing ball model are studied. We show that when dissipation is introduced the structure of the phase space is changed and attractors appear. Increasing the amount of dissipation, the edges of the basins of attraction of an attracting fixed point touch the chaotic attractor. Consequently the chaotic attractor and its basin of attraction are destroyed given place to a transient described by a power law with exponent −2. The parameter-space is also studied and we show that it presents a rich structure with infinite self-similar structures of shrimp-shape.

1. Introduction

The origin of the high energy cosmic rays has intrigued the scientists during the first half of the 20th century. However, in 1949, Enrico Fermi [1], in his pioneering paper *On the origin of cosmic radiation*, proposed that a charged particle could be accelerated by iterations with time-dependent magnetic structures. Since then many different models were proposed in classical [2–9] and in quantum domains [10–14].

One of the models that has been considered very often in the literature is the Fermi–Ulam model (FUM), sometimes called as bouncing ball model. The system consists of a classical particle (denoting the cosmic ray) confined inside and bouncing between two rigid walls: one of them is moving periodically in time (corresponding to the moving magnetic field) while the other is fixed (returning mechanism of the particle towards a further collision with the moving wall). Despite the simplicity of the model, the non-dissipative dynamics of the problem has a very rich and complex phase space. Therefore depending on both the control parameters and initial conditions, regular regions such as invariant spanning curves (also known in theory of nonlinear dynamics as invariant tori) and Kolmogorov–Arnold–Moser (KAM) islands are observed coexisting with chaotic seas. Contrary to what would be expected by the collisions with the moving wall, Lichtenberg and Lieberman [15] showed that the existence of a set of invariant tori in the phase space prevent the particle to accumulate unlimited energy. However, searching for conditions to produce the unlimited energy growth of the bouncing particle, Leonel and Silva [16] in 2008 proposed a specific type of external perturbation of the wall that, depending on the range of control parameters, the growth in the particle's energy was observed. Indeed they considered that the moving wall is connected to a crank by a rod. Therefore for such a system, there are two nonlinearities in the dynamics playing important roles in the velocity of the particle. When the length of the crank approaches the limit of the length of the rod, the motion of the moving wall becomes very fast for certain phases leading the velocity of the wall to become discontinuous for such phases leading to large jumps in the particle velocity and hence to unlimited energy growth. Before such a limit however, the phase space experiences abrupt destruction of invariant tori due to parameter changes leading to merging and overlaps of different chaotic seas.

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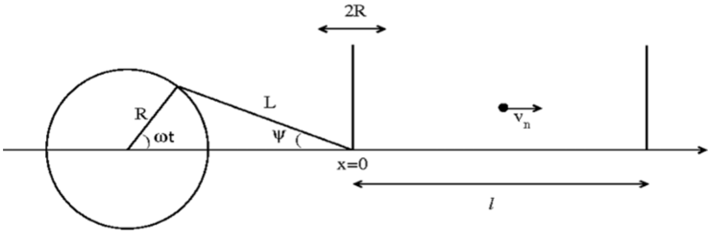

Fig. 1. Illustration of the model.

In this paper we revisit the problem proposed in Ref. [16] seeking to understand and describe some dissipative properties of the dynamics. Thus the model consists of a classical particle confined between two rigid walls. One wall is fixed and the other one moves periodically in time. We assume that the particle experiences inelastic collisions with both walls leading to a fractional loss of energy upon collision. For the fixed wall, we introduce a coefficient $\alpha \in [0, 1]$ while for the periodically varying wall we consider $\beta \in [0, 1]$. The limit $\alpha = \beta = 1$ recovers the results for the non dissipative case. We describe the dynamics via a two dimensional non-linear area-contracting map with four effective control parameters namely: two dissipation parameters and two parameters controlling the nonlinearity of the system. We show that the introduction of dissipation destroys the mixed structure of the phase space. A direct conclusion is that elliptical fixed points turn into sinks and the chaotic sea can be replaced by a chaotic attractor [17]. Each of these structures have their own basin of attraction. When the amount of the dissipation is increased, the edges between the basin of attraction of the attracting fixed point and of the basin of attraction of the chaotic attractor[1] touch the unstable manifold[2] and as a result the chaotic attractor as well as its basin of attraction are suddenly destroyed. Such an event is called boundary crisis [17–19]. After the destruction, the chaotic attractor is replaced by a transient which is characterized by a power law where the independent coordinate is the relative distance, in the control parameter, where the crisis takes place.

Dissipative systems have attracted much attention during the last decades, not only in order to characterize boundary crisis but also because they can be applied in turbulent dynamics [20], molecular physics [21] and many other fields. In the field of Fermi acceleration (FA), it has been shown that the introduction of dissipation works as a mechanism for the suppression of the unlimited energy growth [22–26]. Attention also has been devoted to the parameter space of the dissipative systems. In particular much interest is applied in order to investigate the existence of structures called *shrimps* [27], as reported by Gallas in Ref. [27] in 1993. Since then and considering the advances of fast computers, the parameter-space of dissipative models has received special attention and extensive works have been done not only in theoretical models [28–34] (and references therein) but also very recently the existence of shrimp-shaped domains experimentally involving a circuit of the Nishio–Inaba family [35] has been shown. Here, we have used Lyapunov exponents to classify regions in the parameter-space as regular (null or negative Lyapunov exponent) or chaotic behavior (positive Lyapunov exponent). The procedure we have adopted is: starting with a fixed initial condition, for each increment in the parameters we follow the attractor. This means that we use the last value obtained for the dynamical variables before the increment, as the new initial condition after the increment and we show that the parameter space exhibits a shrimp shade structure which corresponds to the periodic attractors embedded in a chaotic region.

The present paper is organized as follows. In Section 2 we describe all the necessary details to obtain the two-dimensional mapping that describes the dynamics of the system. Moreover, our numerical results for the boundary crisis and shrimp-shape structures are shown along the section. Finally, conclusions are drawn in Section 4.

2. A dissipative Fermi–Ulam model with two nonlinearities

In this section we revisit the model introduced by Leonel and Silva in 2008 [16], however, here, the dissipative dynamics is taken into account. We begin describing the problem and the steps necessary to construct the mapping. The model consists of a classical particle confined between two rigid walls. One of them is considered to be fixed at $x = l$ while the other one is connected in a crank by a rod with length L and it moves periodically in time according to $x_w(t) = R\cos(\omega t) + \sqrt{L^2 - R^2\sin^2(\omega t)} - L$, constructed from the center of the circle, where R is the radius of the crank (the amplitude of oscillation) and ω is the frequency as one can see in Fig. 1. The particle is in the complete absence of any external field. We assume that collisions with both walls are inelastic. Thus, the restitution coefficient $\beta \in [0, 1)$ is introduced when the particle hits the periodically time varying wall. On the other hand, in collisions with the fixed wall, the coefficient $\alpha \in [0, 1)$ is assumed. We take into account values for both α and β inside the interval $(0, 1)$. The dynamics of the system is described in terms of a two dimensional non-linear area-contracting map for the variables velocity and time. Thus, it is convenient to define some dimensionless variables. The time is measured in terms of the number of oscillations of the moving wall,

[1] The edges correspond indeed to the stable and unstable manifolds of a saddle fixed point. They are obtained by using the inverse of the map.

[2] Chaotic attractor.