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Study of a market model with conservative exchanges on complex networks

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ABSTRACT

Many models of market dynamics make use of the idea of conservative wealth exchanges among economic agents. A few years ago an exchange model using extremal dynamics was developed and a very interesting result was obtained: a self-generated minimum wealth or poverty line. On the other hand, the wealth distribution exhibited an exponential shape as a function of the square of the wealth. These results have been obtained both considering exchanges between nearest neighbors or in a mean field scheme. In the present paper we study the effect of distributing the agents on a complex network. We have considered archetypical complex networks: Erdös–Rényi random networks and scale-free networks. The presence of a poverty line with finite wealth is preserved but spatial correlations are important, particularly between the degree of the node and the wealth. We present a detailed study of the correlations, as well as the changes in the Gini coefficient, that measures the inequality, as a function of the type and average degree of the considered networks.

1. Introduction

An empirical study focusing the income distribution of workers, companies and countries was first presented more than a century ago by Italian economist Vilfredo Pareto. He observed, in his book "Cours d'Economie Politique" [1], that the distribution of income does not follow a Gaussian distribution but a power law. That means that the asymptotic behavior of the distribution function follows a power function that decreases, for big values of the wealth, as $w^{-\alpha}$, being $\alpha > 1$ the exponent of the power law. Non-Gaussian distributions are denominated Lévy distributions [2], thus this power law distribution is nowadays known as a Pareto–Lévy Distribution. The exponent α is named the Pareto index. The value of this exponent changes with geography and time, but typical values are close to $3/2$. The bigger the value of the Pareto exponent the higher the inequality in a society.

More recent wealth distribution statistics indicate that, even though Pareto distribution provides a good fit in the high income range, it does not agree with the observed data over the middle and low income ranges. For instance, data from Japan [3], Italy [4], India [5], the United States of America and the United Kingdom [6–8] are fitted by a log-normal or Gaussian distribution with the maximum located at the middle income region plus a power law for the high income strata.

Power laws are not exceptions in nature [9], so it is not surprising that the wealth distribution follows a power law. The quiz with the income and wealth distribution is not the power law, but how this distribution is generated through the

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dynamics of the agents interacting. On the other hand, a Pareto–Lévy distribution is more unequal than a Gaussian: when the distribution follows a power law there are more affluent agents than in the case of a Gaussian distribution, but also more impoverished agents.

In order to try to describe the processes that generate a given profile for the wealth distribution diverse exchange models have been widely applied to describe wealth and/or income distributions in social systems. Different mathematical models of capital exchange among economic agents have been proposed trying to explain these empirical data (for a review see Ref. [10]). Most of these models consider an ensemble of interacting economic agents that exchange a fixed or random amount of a quantity called "wealth". This wealth represents the agents welfare. The exact choice of this quantity is not straightforward, but one can think that it stands for the exchange of a given amount of money against some service or commodity. Within these models the amount of exchanged wealth when two agents interact corresponds to some economic "energy" that may be randomly exchanged. If this exchanged amount corresponds to a random fraction of one of the interacting agents wealth, the resulting wealth distribution is, unsurprisingly, a Gibbs exponential distribution [6] that fits the wealth distribution for the low and middle income range.

Aiming at obtaining distributions with power law tails, several methods have been proposed. Numerical procedures [10–15], as well as some analytical calculations [16,17], indicate that one frequent result of that kind of model is condensation, i.e. concentration of all available wealth in just one or a few agents [18]. This result corresponds to a kind of equipartition of poverty: all agents (except for a set of zero measure) possess zero wealth while few ones concentrate all the resources. In any case, an almost ordered state is obtained, and this is a state of equilibrium, since agents with zero wealth cannot participate in further exchanges. The Gini coefficient [19] of this state is equal to 1, indicating perfect inequality [10,17,18]. Several methods have been proposed to avoid this situation, for instance, exchange rules where the poorer are favored [5,10,15,17,20] but in all circumstances the final state is one with high inequality, i.e. very near condensation.

A few years ago one of us presented an alternative model for wealth distribution, the Conservative Exchange Market Model (CEMM), inspired by the ideas of John Rawls [21] and Amartya Sen [22] and also by the Bak–Sneppen model for extinction of species [23]. The main point of the model is that some kind of action should be taken to change the state of the poorest agent in the society. The idea of a society that takes measures in order to improve the situation of the most impoverished is compatible with the propositions of John Rawls, in his book "A Theory of Justice" [21], directed towards an inventive way of securing equity of opportunities as one of the basic principles of justice. He asserts that *no redistribution of resources within a state can occur unless it benefits the least well-off: and this should be the only way to prevent the stronger (or richer) from overpowering the weaker (or poorer)*. The practical way to carry out this proposition in a simulation was adapted from the Bak–Sneppen model for extinction of biological species [23]. In this model the less fit species disappears and is replaced by a new one with different fitness, then, the appearance of this new species affects the environment changing the fitness of the neighboring species. In 2003 a similar model was developed where the role of the fitness is substituted by the assets of a particular agent, and the model is now conservative, the difference between the new and the old assets of the minimum wealth agent is taken from (if positive) or given to (if negative) the neighbors of the poorest agent [24,25]. The distribution obtained follows an exponential law as a function of the square of the wealth and a poverty line with finite wealth is obtained by self-organization, i.e. the poorer agents possess finite endowments (different to what happens in most exchange models where the minimum wealth is zero). The model was also studied in a kind of mean-field version where the poorest agent interacts with randomly chosen agents [26,15] or by selecting at random as partner of the agent with the minimal value of wealth one of the neighbors of the first agent, and then both agents re-shuffle their entire amount of wealth [27]. Finally, redistribution with the full society was also considered [28]. The obtained Gini coefficient is relatively low and compares well with the values of the Gini coefficient of some Northern European countries as Denmark or Sweden [25]. This suggests a path to decrease inequality in real societies [25,28]. A similar model was presented recently by Ghosh et al. [29] where particles below a given arbitrary threshold may interact with other particles. They studied the system in mean-field, 1*D* and 2*D* and found a distribution that deviates from the exponential Boltzmann–Gibbs one. Also, a phase transition in the number of particles below the poverty line is found as a function of the value of the threshold. This phase transition is not present in the CEMM theory because here the threshold is a result of the self-organization induced by the extremal dynamics.

Here we revisit this model considering a more realistic description of social networks. We study the system on scale-free network (SF) and on random Erdös–Rényi (ER) network (for a review on these networks see, for example, Ref. [30]). The case of Watts–Strogatz small world networks has already been partially discussed in Ref. [15] and the results do not differ in a significant way from the mean field results of ref. [24]. Besides, we do not try to describe the power law region of the wealth distribution, as we are only considering additive exchanges. The results will describe the middle and low income agents, and also, we will obtain a poverty line that is a characteristic of some countries, like Scandinavian ones where protective measures are taken to help the less favored people. Anticipating the results we are going to show that the poverty line is a robust characteristic of the model, but a strong correlation between the connectivity of the agents and their wealth is also observed. Indeed, it seems that the more connected agents exhibit a wealth less than average, because they have higher probability of interacting with poorer agents.

In the next section we present a very short review of the original model: the Conservative Exchange Market Model (CEMM) and its main conclusions. Then, in Section 3 we introduce the SF and ER networks and present the main results for these networks. In Section 4 we discuss the results and present the conclusions.