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Electric diffusion in cylindrical conductors from an extended irreversible thermodynamics perspective

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ABSTRACT

Inertial effects on electric diffusion are explored in cylindrical conductors at the quasi-static limit. Such effects are described by introducing a finite relaxation time for the current density. When the electric field penetrates deeply into the conductor, it is shown that the surface inductance attains a minimum, L_m, if the radius of the cylinder acquires a critical value, a_c. The proposed formulation is applied to aluminum at room temperature, and it is found that $L_m \sim 22.1$ fH for $a_c \sim 35.1$ nm. This shows that inertial effects should be important at the nanoscale. The results presented here may be relevant for investigations in nanophotonics.

1. Introduction

The phenomenon of the so-called skin effect, the confinement of electric and magnetic fields to a thin layer, close to the surface of a good conductor, as a limiting case of the more general process of electromagnetic diffusion, at the quasi-static approximation (the displacement current is neglected), has been the object of intensive research, both experimental and theoretical, for a long time [1–8]. Renewed interest in the study of the skin effect, occurring in cylindrical conductors, has been invoked in connection with practical problems of transmission lines, such as attenuation and distortion of electromagnetic signals, for both homogeneous and non-homogeneous media [9–11].

The opposite limiting situation of electromagnetic diffusion, for which the fields penetrate deeply into the conductor, has been examined, also, for cylindrical structures [12,13]. Recent investigations in the subject have been motivated by the possibility of optically manipulating power flows in photonic nanowires [14–16].

In the classical approach to the problem, the induced current density $\vec{J}$ is related to the applied electric field $\vec{E}$ through the standard Ohm's law expression,

$$\vec{J} = \sigma \vec{E}, \tag{1}$$

where σ is the electric conductivity. Such an expression is widely known to describe satisfactorily the dissipative processes which occur at sufficiently low frequencies. Actually, if the field were suddenly turned off, Eq. (1) predicts that the current would vanish instantaneously. This shows that the ordinary Ohm's law ignores inertial effects due to charge carriers.

Inertial effects may be included in the discussion on the basis of extended irreversible thermodynamics. This approach treats the dissipation function of linear processes as the average of the statistically fluctuating dissipation rate on either coarse or small spatial scales [17–20]. In recent years, a link has been established between such a formulation and the possibility of extending some constitutive relations of matter to describe dissipative processes occurring at sufficiently fast rates [21–23].

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In the framework of extended irreversible thermodynamics, Eq. (1) may be generalized to

$$\left(1+\tau\frac{\partial}{\partial t}\right)\vec{J}=\sigma\vec{E}, \tag{2}$$

where τ is a constant with dimension of time. Actually, in the limit $\tau \to 0$, Eq. (2) recovers Eq. (1). Now, if the electric field is suddenly removed from the conductor, Eq. (2) leads to

$$\vec{J}=\vec{J}_0\mathrm{e}^{-t/\tau}. \tag{3}$$

This shows that any initial current density $\vec{J}_0$ dissipates in the conductor in a time scale of the order of τ. Therefore, τ may be physically interpreted as the relaxation time of the current density, which is driven by inertial effects of charge carriers at sufficiently high frequencies.

In previous work, the attenuation and damping of electromagnetic fields have been discussed on the basis of Eq. (2) [24]. Subsequently, the skin effect and related dissipative processes were examined for conductors of arbitrary shape [25]. Recently, an extension of the Hagen–Rubens relation was obtained up to the near-infrared [26]. More recently, the phenomena of resistive heating and magnetic diffusion have been investigated for good conductors [27].

This work explores inertial effects of charge carriers on the process of electric diffusion in cylindrical conductors at the quasi-static limit on the basis of Eq. (2). The two limiting situations for electric diffusion, namely, the skin effect and field penetration, are examined.

2. Preliminary considerations

Before we embark on the subject of electric diffusion properly, let us see, in some detail, two immediate consequences of including the inertial term on the left-hand side of Eq. (2). These are corrections to the relaxation process of free charges and the quasi-static limit of so-called good conductors.

2.1. Charge relaxation

We start by considering the continuity equation for free charges,

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}, \tag{4}$$

where ρ is the charge density. Then, the combination of Eqs. (4) and (2) leads to

$$\tau\frac{\partial^2\rho}{\partial t^2}+\frac{\partial\rho}{\partial t}+\frac{\sigma}{\epsilon}\rho=0, \tag{5}$$

where use has been made of Gauss's law,

$$\nabla\cdot\vec{E}=\frac{\rho}{\epsilon}, \tag{6}$$

with ϵ denoting the electric permittivity of the conductive medium. To the best of our knowledge, Eq. (5) has not been previously derived in the literature. Now, if we regard a fixed, sufficiently small volume V_0 within the bulk of the conductor, the instantaneous total charge flowing through it is given by

$$q\left(t\right)=\int_{V_0}\rho dV. \tag{7}$$

Thus, in view of Eq. (7), it follows from Eq. (5) that the time evolution of the local charge within the conductor is governed by

$$\tau\frac{d^2q}{dt^2}+\frac{dq}{dt}+\frac{\sigma}{\epsilon}q=0. \tag{8}$$

The general solution of Eq. (8) depends on the comparison of two time scales, namely ϵ/σ and τ.

If $\tau<\epsilon/\sigma$, the instantaneous charge is given by

$$q\left(t\right)=q_0\left(\frac{\gamma_+\mathrm{e}^{-\gamma_-t}-\gamma_-\mathrm{e}^{-\gamma_+t}}{\gamma_+-\gamma_-}\right)+\frac{I_0}{\gamma_+-\gamma_-}\left(\mathrm{e}^{-\gamma_-t}-\mathrm{e}^{-\gamma_+t}\right), \tag{9}$$

where the fast (plus sign) and slow (minus sign) damping rates are

$$\gamma_\pm=\frac{1\pm\sqrt{1-4\sigma\tau/\epsilon}}{2\tau}, \tag{10}$$