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Analysis of phase transition points for a two-color agent-based model

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ABSTRACT

The agent-based model treated in the present study describes dynamics of two types of population in a gravity-like potential field. In previous studies, the model was known to exhibit various spatiotemporal patterns on two-dimensioanl lattice systems. However, the patterns were classified depending purely on eye observations, and the underlying dynamics of these patterns were not fully explored. It remained a question to be answered if these eye observation-based classifications could be confirmed by any analytical means. To pursue the question, we first suggest several analytic quantities, such as convergence time steps and reaction speed, to replace the eye observations. As a result, we show that a phase diagram can be reasonably drawn on the contour diagram of the time steps. In addition, we find a power-law scaling in the reaction speed, confirming that a phase transition really is involved there. Next, as a main part of the present study, we apply analytical methods to calculate two important phase transition points from the system. The results from the analytical approach agreed well with the numerically obtained phase transition points from the agent-based model. In general, the paper serves as an example study of estimating global phenomena of complex systems in terms of local parameters of the system.

1. Introduction

There are many systems in nature and society composed of heterogeneous groups of individuals. An ecological system composed of predator–prey or host–parasitoid species and a city composed of multiple races may be among the examples. Scientific studies of these heterogeneous systems are frequently concerned with finding or explaining the emergence of global patterns out of simple interactions between the agents. For example, theoretical ecologists try to find a model that can explain the persistent coexistence of the two species, predator and prey, without going into extinction [1,2]. Sociologists try to identify the underlying mechanism that can explain residential segregation commonly found in metropolitan cities of, for example, the US [3–5].

The agent-based model treated in the present study is also concerned with such a two-group problem. The system is composed of two types of agent distributed on a square grid system and moving with discrete time steps [6]. The dynamics are defined in terms of gravity-like potential and the agents move to improve their potential. Though not designed for a specific system, the model was successfully applied to explain real-world phenomena. For example, the initial monochrome version of the model was applied to explain the power-law size distribution of the cities in a country [7]. And a version of the two-color model was applied to simulate residential segregation phenomena in urban areas [8] and to simulate sexual segregation phenomena in an ecological system [6,9].

In its concept, the present model is comparable to swarm chemistry [10]. In swarm chemistry, a wide variety of spatiotemporal patterns of collective behavior emerge through the kinetic interaction between the heterogeneous agents

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representing multiple chemical species. The reaction rules in swarm chemistry are meant to be realistic, and, as a result, the dynamics of swarm chemistry is rather complex, including a large number of parameters. In contrast, the model treated in the present study is relatively simple, with just one rule, which is defined in terms of just one set of parameters.

In spite of its simple dynamics, the present model revealed a wide variety of static/dynamic patterns. A phase diagram was suggested to distinguish between different modes, or types of pattern, emerging from the system [6]. The patterns were classified as static or dynamic, depending on the time steps required for the system to come to a fixed state. Dynamic patterns were further identified into various modes depending on the shapes of the patterns. Among the modes, an interesting *Cell* mode was observed which reminded us of the cell division process in a biological system. In the previous study, however, eye inspection of the patterns on a computer screen was the only means to identify different types of pattern. In a practical sense this may be sufficient. But to be more scientific, we were in need of an analytical means to replace the eye inspection. In particular, it was of theoretical interest to see if the observed patterns could be classified on some quantitative basis. In addition, we wanted to explain the mechanism of the phase transitions, if any, between different modes. For the purpose of answering these questions, in the present paper, we focus on the most interesting mode of the system, the *Cell* mode. We first show that the *Cell* mode can be differentiated quantitatively from the neighboring modes and that the *Cell* mode appears at a phase transition point between dynamic and static modes. As a main part of the paper, we explain the mechanism by which the phase transitions occur and finally analyze the phase transition points theoretically.

2. Two-population agent-based model

2.1. Model description [6]

The agents in the present model are defined on a two-dimensional square lattice system composed of $H \times H$ cells. Throughout the present study, we use periodic boundary conditions. Two type of agent, type A and type B, are distributed on the system in such a way that only one agent can occupy a cell. The number of agents is assumed to be $N_{\mathrm{A}} = N_{\mathrm{B}} = N/2$, where N is the total number of agents. An agent i has a potential defined as follows:

$$\Phi_i = \sum_{j=1 \text{ to } N}^{j \neq i} \frac{\alpha_{\mathrm{IJ}}}{d_{ij}}. \tag{1}$$

In Eq. (1), $I \in \{\mathrm{A}, \mathrm{B}\}$ denotes agent i's type. The summation is carried over all agents in the system except itself. The variable d_{ij} measures the Euclidean distance between agents i and j. The parameter α_{IJ} is called the homophily constant; it comes into effect when agent i of type I and agent j of type J interact. We have four homophily constants corresponding to interactions between and within the two types of agent, namely α_{AA}, α_{AB}, α_{BA}, and α_{BB}. The potential defined in Eq. (1) is similar to gravitational potential, the universal constant being differently defined between different types of agent and the mass of the individual agents being set to 1.0.

We specify next the condition under which migration occurs. The key behavioral assumption here is agents' desire to find locations that yield higher potentials. However, since a site's maximum carrying capacity is one agent at any given time, and since we do not allow swapping between agents, the only possible relocation of an agent is into an empty site. Consider the case in which agent a evaluates the attractiveness of empty site e. That agent migrates to empty site e if the following condition is satisfied:

$$\Phi_e - \Phi_a \geq f d_{ae}, \tag{2}$$

where f is a friction factor and d_{ae} is the distance between agent a's current location and the empty site e. The potential Φ_e denotes the potential an agent might have if it moves into the empty cell e. The right-hand term $f\,d_{ae}$ corresponds to the moving cost, which is proportional to the distance between the agent's origin location and its destination. Thus, according to condition (2), an agent relocates if its increase in potential is greater than the moving cost. The system starts from an initial random distribution. At each time step, an agent and an empty cell are chosen at random. If the move condition (2) is satisfied, then the agent migrates into the empty cell. If not, no action takes place. There can be at most one migration in a time step and more frequently there is no action in a time step.

2.2. Phase diagram

2.2.1. Phase diagram based on convergence time steps

The behavior of the system depends on the homophily constants and the friction factor. In the present paper, we reduce the space of parameters introducing normalizations on the parameters such as $\alpha_{\mathrm{AA}} + \alpha_{\mathrm{AB}} = 1$ and $\alpha_{\mathrm{BA}} + \alpha_{\mathrm{BB}} = 1$. Denoting two independent homophily constants as $\alpha \equiv \alpha_{\mathrm{AA}}$ and $\beta \equiv \alpha_{\mathrm{BB}}$, we have the following set of homophily constants:

$$\{\alpha_{\mathrm{AA}}, \alpha_{\mathrm{AB}}, \alpha_{\mathrm{BA}}, \alpha_{\mathrm{BB}}\} = \{\alpha, 1-\alpha, 1-\beta, \beta\}. \tag{3}$$

In addition, we restrict the present study to the intervals $1.0 \leq \alpha \leq 2.0$ and $0.0 \leq \beta \leq 1.0$. The condition $1.0 \leq \alpha \leq 2.0$ gives $1.0 \leq \alpha_{\mathrm{AA}} \leq 2.0$ and $-1.0 \leq \alpha_{\mathrm{AB}} \leq 0$. This means that an agent of type A likes same type agents but dislikes different