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Structure properties of evolutionary spatially embedded networks

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ABSTRACT

This work is a modeling of evolutionary networks embedded in one or two dimensional configuration space. The evolution is based on two attachments depending on degree and spatial distance. The probability for a new node n to connect with a previous node i at distance r_{ni} follows $a\frac{k_i}{\sum_j k_j} + (1-a)\frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}^{-\alpha}}$, where k_i is the degree of node i, α and a are tunable parameters. In spatial driven model ($a = 0$), the spatial distance distribution follows the power-law feature. The mean topological distance l and the clustering coefficient C exhibit phase transitions at same critical values of α which change with the dimensionality d of the embedding space. When $a \neq 0$, the degree distribution follows the "shifted power law" (SPL) which interpolates between exponential and scale-free distributions depending on the value of a.

1. Introduction

In the recent development of network sciences [1–9], spatial constraint networks have become an object of extensive investigation [9–23]. These are the networks embedded in configuration space and influenced by spatial constraints. Recent findings have revealed that the spatial distance distribution follows power law or exponential distribution [6,7,9,17,18]. These distributions are quite natural since, for instance, people tend to have their friends and relatives in their neighborhood, transportation networks often favor shorter distance trips, and many communication networks are mainly dominated by short radio ranges [20]. To model these systems, scientists have proposed spatially constrained networks embedded in one- or two-dimensional space [10–13,15,19,21,22]. According to the generation rules, these networks can be categorized into three classes: scale-free (SF) networks with disadvantaged long-range links, SF networks embedded in lattices and space-filling networks [18].

The first class is an extension of the conventional SF models by adding competition between degree and spatial distance preferences of linking. In one of the extended models, the network grows with addition of nodes randomly positioned in space. The nodes are connected to each other with the probability $\Pi_i \sim k_i r^{\alpha}$, where r is the spatial distance between the new node and the node i with degree k_i. The distance distribution $p(r)$ is given by $p(r) \sim r^{-(\alpha-d+1)}$ as expected, where d is the dimension of the space. On the other hand, the degree distribution is a power law for $\alpha > -1$ and a stretched exponential law for $\alpha < -1$ [15,19]. Another extension uses the connection probability $\Pi_i \sim k_i^{\beta} r^{\alpha}$ and generates a power law degree distribution on a line in the $\alpha - \beta$ plane and in the zone limited by $\beta > 1$ and $\alpha < -0.5$ [11]. The second class is

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an extension of the SF networks by embedding them in regular one- or two-dimensional spatial lattices in which the links are added according to the probability $p(r) \sim r^{-\delta}$. The structure of the network is affected by the parameter δ [13]. The third class uses the method of space-filling packing in which a region is iteratively partitioned into subregions by adding new nodes which are connected to the closest neighbors [10,12,21,22].

In this work we are interested in the first class networks. We introduce a model with a kind of competition between the degree and the spatial distance preferences. While the degree preferential attachment produces connections free from spatial constraints, the spatial distance preference favors closer connections. This competition between short-range and long-range connections is modulated by a parameter a.

2. The model

To construct the networks, the nodes are embedded on a one-dimensional ($d = 1$) ring of radius $R = 1/\pi$ or on a two-dimensional ($d = 2$) sphere of radius $R = 1/\pi$. The spatial distance r between a pair of nodes is defined as the shortest distance between them.

The model is constructed in the following way:

(1) *Initial condition*: We start with an initial state ($t = m_0$) of $m_0 + 1$ all-to-all connected nodes on the ring or the sphere.
(2) *Growth*: At every time step, a new node is added, which is randomly placed on the ring or the sphere.
(3) *Addition of edges*: The new node n connects with $m(m \leq m_0+1)$ previous nodes, which are selected with the probability π_i

$$\pi_i = a\frac{k_i}{\sum_j k_j} + (1-a)\frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}{}^{-\alpha}} \tag{1}$$

where k_i is the degree of node i, r_{ni} is the Euclidean distance between a new node n and a previous node i, $0 \leqslant a \leqslant 1$ and $0 \leqslant \alpha$. The growing process repeats step (2) and (3) until the network reaches the desired size. Accordingly, at each step, the number of nodes increases by one, while the number of edges increases by m ($m = m_0 = 2$ in what follows if not mentioned). Hence at time t, the network contains $t + 1$ nodes and $m(t + 1)$ edges.

This model has two limit cases: when $a = 1$, the network recovers the SF network model, while the case of $a = 0$ and $\alpha = 0$ corresponds to the random growing process.

The numerical results described in this paper are the average of 20 simulations for different realization of networks under the same parameters with the network size of 10 000 nodes. We have also tried 50 000 nodes, but the result is almost the same.

3. Spatial driven model

In this section, we focus on the behavior of pure spatial-driven model with $a = 0$ and $\alpha \neq 0$ in Eq. (1). The connection probability is

$$\pi_i = \frac{r_{ni}^{-\alpha}}{\sum_j r_{nj}{}^{-\alpha}}. \tag{2}$$

3.1. Degree distribution

The nodes are labeled by their birth times, $s = 0, 1, 2 \cdots t$. $p(k, s, t)$ is the probability that the node s has degree k at time t. The master equation of $p(k, s, t)$ is given by

$$p(k, s, t+1) = \frac{m}{t+1}p(k-1, s, t) + \left(1 - \frac{m}{t+1}\right)p(k, s, t). \tag{3}$$

The initial conditions are $p(k, s = 0, 1 \ldots m_0, t = m_0) = \delta_{k,m_0}$ and $p(k, t, t) = \delta_{k,m}$.$p(k, s, t + 1)$ contains two parts. The first one comes from the nodes having degree $k - 1$ at time t and selected to connect with the new node at time $t + 1$. The second one comes from the nodes having degree k at time t and not selected at time $t + 1$.

The degree distribution of the entire network can be written as

$$p(k, t) = \frac{1}{t+1}\sum_{s=0}^{t} p(k, s, t). \tag{4}$$