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Heat baths and computational agent-based models

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ABSTRACT

In this paper, we examine an agent-based model, and an equation-based model in the form of a mean field model. We show how the mean field model is a small, fast model that identifies the high level properties of a subject, in this case financial time series' stylized facts. The agent based model generates the granularity needed to understand the conditions and factors that generate the stylized financial facts. We conclude with the recommendation that both models be used in sequence so a complete description of a process be established or approximated.

1. Computational agent-based models

1.1. The evolutionary dynamics of adaptive belief systems

For agent-based models this paper focuses on the adaptive rational equilibrium modeling first developed by Brock and Hommes [1,2] and used by Chiarella and He [3]. Brock and Hommes (hereafter, B & H) developed what they call "adaptive belief systems" to model heterogeneous expectations in a way that naturally mimics real market activity.

In adaptive belief systems agents (investors) adapt their prediction of asset prices by choosing a finite number of predictors or expectation functions, which are a function of past price performance. Each predictor has a performance measure that is visible to all participants. Based on the performance measure, agents make a (bounded) rational choice between price predictors. This assumption of bounded rational choice results in an adaptive rational equilibrium dynamic (ARED), which generates a dynamic across predictor choices that is coupled to the dynamics of the endogenous variables. B & H [1] show that ARED incorporates a general mechanism that can generate local instability in the equilibrium steady state and complicated global equilibrium dynamics.

B & H apply the concept of adaptive belief systems to a simple asset-pricing model, where traders in a financial market use different types of predictors for the price forecasts of risky assets. Chiarella and He [3] extend B & H's work to include agents with different risk aversion as well as expected dividend payments with noise. What follows in this section is the initial setup of an adaptive belief system as used by Chiarella and He (hereafter, C & H).

To define investor wealth it is assumed the risk-free asset is perfectly elastic and supplied at a gross return $R > 1$. Let p_t be the price (ex dividend) per share of the risky asset at time t, and let (y_t) be the stochastic dividend process of the risky asset. Then, investor wealth at $t + 1$ is defined as:

$$W_{t+1} = RW_t + (p_{t+1} + y_{t+1} - Rp_t)\, z_t \tag{1.1}$$

where Rp_t is the return of the risk-free asset at time t, and z_t is the number of shares of the risky asset purchased at time t.

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In both B & H's and C & H's work a Walrasian auctioneer is used to derive the demand equation, i.e., each trader is viewed as a price taker. The market is viewed as finding the price p_t that equates the sum of the demand schedules to the supply. That is, the price p_t at time t is formed by using information available at time $t-1$ and the expected utility for time $t+1$.

Denote by:

$$F_t = (p_t, p_{t-1}, \ldots, y_t, y_{t-1}, \ldots) \tag{1.2}$$

the information set formed at time t, where p is the price that equates the sum of the demand schedule to the supply of the asset and y is the dividend yield.

Now, denote by R_{t+1} the excess return on the risky asset at time $t+1$:

$$R_{t+1} = (p_{t+1} + y_{t+1} - Rp_t)\,. \tag{1.3}$$

It follows from Eqs. (1.2) and (1.3) that the conditional expectation E_t and conditional variance V_t can be defined as:

$$\begin{aligned} E_{ht}(W_{t+1}) &= RW_t + E_{ht}(p_{t+1} + y_{t+1} - Rp_t)\, z_t \\ &= RW_t + E_{ht}(R_{t+1}) z_t \end{aligned} \tag{1.4}$$

$$V_{ht}(W_{t+1}) = z_t^2 V_{ht}(p_{t+1} + y_{t+1} - Rp_t) = z_t^2 V_{ht}(R_{t+1}) \tag{1.5}$$

where the subscript h refers to the beliefs of investor type h about the conditional mean and variance.

Note that in Eq. (1.5) the conditional variance of wealth W_{t+1}equals z_t^2 times the conditional variance of excess return per share $p_{t+1} + y_{t+1} - Rp_t$. This differs from B & H [2], who assume the beliefs about conditional variance of excess returns are constant and the same for everyone, i.e., $V_{ht}(p_{t+1} + y_{t+1} - Rp_t) \equiv \sigma^2$.

C & H assume each type of investor is a mean-variance maximizer with different attitudes toward risk. Each investor type h has a risk aversion coefficient a_h. Given a_h, for each investor type the demand for shares z_{ht} solves:

$$\max_z = \left(E_{h,t}(W_{t+1}) - \frac{a_h}{2} V_{h,t}(W_{t+1}) \right) \tag{1.6}$$

or

$$z_{h,t} = \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}. \tag{1.7}$$

Let z_{st} be the supply of shares and n_{ht} the fraction of type h investors at t. Using Eq. (1.7), the equilibrium state of supply equaling demand is described by:

$$\sum_h n_{h,t-1} \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})} = z_{st}. \tag{1.8}$$

Now, assume a zero supply of outside shares, $z_{st} = 0$, then (1.8) leads to:

$$\sum_h n_{h,t-1} \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})} = 0. \tag{1.9}$$

Eq. (1.9) makes it appear that the effect of the risk-aversion coefficients is to rescale the $n_{h,t-1}$ functions. This will be found to not be the case once Eq. (1.12) below is introduced.

To get a notion of the rational expectation (RE) fundamental solution p_t^*, consider the equation:

$$Rp_t^* = E_t(p_{t+1}^* + y_{t+1})$$

where E_t is the expectation conditional on the information set F_t (see Eq. (1.2) above). To satisfy the "no-bubbles" version of the rational expectations B & H and C & H are using, the only solution can be:

$$\bar{p} = \frac{\bar{y}}{R+1}.$$

If we let x_t denote the deviation of p_t from the RE fundamental p_t^*, then:

$$x_t = p_t - p_t^*.$$

After establishing this relationship C & H give the equations for heterogeneous beliefs for returns and variance, i.e., the different classes of beliefs that are deviations from the fundamental.

Both B & H and C & H use a fitness function, which is defined by the realized profits of investor type h:

$$\begin{aligned} \pi_{h,t} &= R_{t+1} z_{h,t} \\ z_{h,t} &= \frac{E_{h,t}(R_{t+1})}{a_h V_{h,t}(R_{t+1})}. \end{aligned} \tag{1.10}$$

Memory can be introduced to the fitness function by considering a weighted average of past profits:

$$U_{h,t} = \pi_{h,t} + \eta U_{h,t-1} \tag{1.11}$$

where η represents the memory strength.