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Understanding agent-based models of financial markets: A bottom–up approach based on order parameters and phase diagrams

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ABSTRACT

We describe a bottom–up framework, based on the identification of appropriate order parameters and determination of phase diagrams, for understanding progressively refined agent-based models and simulations of financial markets. We illustrate this framework by starting with a deterministic toy model, whereby N independent traders buy and sell M stocks through an order book that acts as a clearing house. The price of a stock increases whenever it is bought and decreases whenever it is sold. Price changes are updated by the order book before the next transaction takes place. In this deterministic model, all traders based their buy decisions on a call utility function, and all their sell decisions on a put utility function. We then make the agent-based model more realistic, by either having a fraction f_b of traders buy a random stock on offer, or a fraction f_s of traders sell a random stock in their portfolio. Based on our simulations, we find that it is possible to identify useful order parameters from the steady-state price distributions of all three models. Using these order parameters as a guide, we find three phases: (i) the dead market; (ii) the boom market; and (iii) the jammed market in the phase diagram of the deterministic model. Comparing the phase diagrams of the stochastic models against that of the deterministic model, we realize that the primary effect of stochasticity is to eliminate the dead market phase.

1. Motivation

Economies and financial markets are complex systems described by a large number of variables, which are in turn influenced by an even larger number of factors or players. To understand how these complex systems evolve in time, an approach that has been very fruitful thus far is to consider the dynamics of a small number of aggregate collections of homogeneous variables. In this approach, the dependences of aggregate averages on other aggregate averages, and on time, are modelled by coupled systems of ordinary or partial differential equations. While this equation-based approach has been able to provide rigorous theorems, and generate much economic insight, it is fundamentally mean field in character, in that variances and higher-order statistical moments of the aggregates are assumed to be slaves of the averages, and have no independent dynamics of their own. In many important and interesting situations in the real world, this assumption is indeed valid, because the number of variables in each homogeneous aggregate is large, and the central limit theorem applies.

However, in many other interesting real-world situations, statistical fluctuations can become an important driver in the time evolution of economies and financial markets. When this happens, the variances and higher statistical moments of the aggregates become large, and their dynamics cannot be deduced from those of the averages alone. This is where agent-based models and simulations become invaluable as a tool for understanding the dynamics of the economic or financial

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system as a whole. Since the pioneering work of Arthur and co-workers [1], there has been a rapidly growing interest in the use of agent-based simulations as a computational platform for performing 'controlled experiments' in an economic setting [2–4]. This has culminated in several reviews [5–7] and monographs [8–10] on the subject. In general, economists have taken a top–down approach to agent-based modelling, implementing neo-classical economic axioms, which are then systematically relaxed to incorporate the effects of heterogeneity [11,12], long-term memory [13], and learning [14]. While this line of agent-based research has been able to give stylized and qualitative results, it is generally very difficult to draw quantitative, and deeper inferences and insights, because of large statistical fluctuations within the simulations.

In contrast, highly simplified models have been the choice of physicists [15–21], because such models are easy to understand in quantitative terms. In this paper, we propose a bottom–up framework for going between the two agent-based modelling approaches. We start by describing in Section 2 a highly simplistic agent-based model, whose purpose is to illustrate the ideas behind the model differentiation framework, rather than to accurately describe the real world. After working out its phase diagram in Section 3, we refine the model by introducing a stochastic component in the trading strategies of the agents. By examining how the phase diagram is modified by the additional rules, we report insights gained from our preliminary study. In Section 4, we discuss how we can refine the model progressively to make its dynamics more and more realistic, and ultimately develop a picture on the hierarchy of complexities that emerge at various levels of realism in the models. Our goal is to eventually be able to identify robust market behaviours, which depend on the gross structure of the models but not the details, and also fragile market behaviours, which depends sensitively on certain model details, and hence are expected to appear only rarely in the real world.

2. Deterministic model

To illustrate the basic framework in our bottom–up approach to understanding agent-based models, we start from a highly simplified model of a financial market, which consists of N traders, M stocks, and an order book. At the start of each simulation, we assign each of the M stocks a random price $0 < p \leq p_{\max}$, and a zero initial price change $\Delta p = 0$. A random initial offer $0 < q \leq q_{\max}$ for each stock is also made available on the order book, which plays the role of a clearing house. We then assign to each of the N traders a random initial capital $c = c_{\max}$, as well as a random portfolio that excludes short positions. In this simple model, we assume that the N traders do not directly interact with each other, but carry out transactions only through the order book. At each time step, the N traders will trade in a randomized sequence. Each trader will buy one stock offered by the order book, and then sell one stock in his or her portfolio.

When it is trader i's turn to trade, he or she will evaluate the utilities $(u_1, \ldots, u_M)$ of the M stocks on offer in the order book, based on the *call function*

$$u_s = \frac{\alpha}{p_s} + \Delta p_s, \tag{1}$$

where p_s is the price of stock s, and Δp_s is the last price change of stock s. Trader i then buys however many units he or she can afford of the stock s^* having the maximum utility. After this transaction, the order book increases the price p_{s^*} of stock s^* by one unit, and sets $\Delta p_{s^*} = +1$. Once this price adjustment is completed, trader i evaluates the utilities $(v_1, \ldots, v_M)$ of all M stocks, using the *put function*

$$v_s = p_s - \beta \Delta p_s. \tag{2}$$

If stock s^{**} is found to have the maximum utility, trader i will sell all of stock s^{**} in his or her portfolio. The order book then decreases the price $p_{s^{**}}$ of stock s^{**} by one unit, and sets $\Delta p_{s^{**}} = -1$ before the next trader trades.

In this model, we introduce two utility functions $u(p, \Delta p)$ and $v(p, \Delta p)$ to accommodate potential asymmetry between seller and buyer trading strategies. In these two utility functions, we also attempt to incorporate both fundamentalist and chartist tendencies. Given two stocks in the order book with the same positive long-term ratings, we expect fundamentalist traders to always buy the cheaper of the two. This tendency is accounted for in the call function by the price term α/p, which decreases with increasing price p. Chartist traders, on the other hand, will only buy an appreciating stock. Hence our choice of the price change term Δp in the call function. We also assume that stocks are merely tools to increase capital assets, and traders attach no further value to them. Therefore, for the purpose of generating cash flow, we expect a fundamentalist trader will always sell his or her most expensive stock. This tendency is modelled by the price term p in the put function. A chartist trader, whose trading decisions are based entirely on price changes, will only sell depreciating stocks. Hence our choice of the price change term $-\beta\Delta p$ in the put function. The traders in our simulations, who have no memory (and thus no capacity to learn), exhibit these fundamentalist and chartist tendencies to different extents, depending on the two independent parameters α and β. We vary α and β to determine the phase diagram of this model, whereby all traders behave rationally, and based their trading decisions on two deterministic utility functions.

3. Order parameters and phase diagram

All our simulations were done with $N = 10{,}000$, $M = 1000$, $c_{\max} = 100$, $p_{\max} = 100$, and $q_{\max} = 100$. We chose these fixed parameters so that our system of agents resemble, at a very gross level, small markets like the Singapore Exchange