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Effect of noise on chaos synchronization in time-delayed systems: Numerical and experimental observations

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HIGHLIGHTS

- We address the effect of colored noise on the synchronization of nonlinear delay systems.
- A detailed analysis of a delay differential equation under colored noise is presented.
- A modified form of the second-order Runge–Kutta method for delay differential equations under stochastic perturbation is devised.
- Lyapunov exponents for a delay differential equation under stochastic forcing are calculated.
- An electronic circuit is used to test the theory experimentally.

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ABSTRACT

We discuss the constructive role of noise (white and colored) in chaos synchronization in time-delayed systems. We first numerically investigate noise-induced synchronization (NIS) between two identical uncoupled Ikeda and Mackey–Glass systems. We find that synchronization occurs above a critical noise intensity that differs for different colors of noise. Synchronization onset is characterized by the value of the maximum transverse Lyapunov exponent. We then discuss the enhancement of chaos synchronization between two time-delayed systems when they are coupled unidirectionally. The effect of parameter mismatch for NIS is described in detail. We provide experimental evidence of NIS for a Mackey–Glass-like system in an electronic circuit using different colors of noise. An integration scheme for time-delayed systems in the presence of additive white and colored noise is discussed.

1. Introduction

According to the conventional concept of chaos synchronization, two or more oscillators that interact with each other follow the same behavior after some time when they exceed a critical coupling. What if the oscillators do not interact with each other? This type of behavior is seen in biological science, neural networks, and laser science [1,2]. Many situations in nature arise in which two or more oscillators interact with each other via a common medium or common external noise. There has been recent interest in synchronization among non-interacting limit cycles [3,4] and chaotic oscillators [5,6] induced by a common fluctuating external force. The strong link found between synchronization and external noise has been termed noise-induced synchronization (NIS) [7–9]. In NIS, a common external noise input to two independent systems can give rise to synchronization regimes that are not observed in the corresponding noise-free system. Numerical and experimental studies on NIS have been conducted for physical systems such as lasers, electronic circuits, biological

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systems, and neuronal networks [10,11]. In neural systems, NIS was first observed between a pair of uncoupled sensory neurons [10]. In Ecology, it is known that, due to common climate fluctuations, populations of plants exhibit large-scale synchronized flowering and production of seed crops also fluctuate synchronously from year to year [11]. When two systems are coupled, the effect of noise on synchronization is already studied in low dimensional systems [12].

Noise is ubiquitous in natural and man-made systems and can play a constructive or a destructive role. Zhou and Kurths found that noise plays a constructive role in enhancing phase synchronization in non-identical systems [13]. This was experimentally verified for electrochemical oscillators [14]. The constructive role of noise has been studied extensively in the context of stochastic resonance [15–17] and coherent resonance [18,19]. The destructive effect of noise has mostly been studied as a robustness test for synchronization since noise is always present in any experiment and in nature [20,21]. It was analytically shown that common NIS can occur in limit cycle systems [22] and numerically shown for chaotic maps [23] and chaotic systems [13]. To date, studies of NIS and noise-enhanced synchronization (NES) [24–27] have investigated limit cycles, chaotic maps, and low-dimensional chaotic systems without a time delay. Most previous NIS studies used white noise, which has infinite variance and no time correlation. Researchers have considered colored noise for cases in which time correlation is important. However, our interest here is in red and green noise because they are positively and negatively correlated, respectively. Little attention has been paid to synchrony induced or enhanced by colored noise in time-delayed systems. The effect of colored noise has been discussed for fixed point, [2], limit cycle [28], and chaotic systems [28]. Delayed dynamic systems have two properties that make studies interesting. First, a deterministic delayed dynamic system has a nonzero memory, that is, it does not satisfy the Markov property. Second, a nonlinear dynamic system with a fixed time delay τ gives rise to infinite-dimensional dynamics on the phase space $C(-\tau, 0)$ of continuous functions on the interval $(-\tau, 0)$. Thus, they are associated with high-dimensional chaotic attractors.

Here we extend NIS and NES studies to the case of delayed dynamic systems driven by common noise. The need for such a study arises from two simple facts. Since a delayed system has no Markovian property, addition of noise or a stochastic perturbation does not change the dynamics to Markovian dynamics. Therefore, well-established methods for Markovian processes, such as the Fokker–Planck equation, cannot be used. Moreover, it is not very clear how small addition of noise can have any significant effect for a higher-dimensional attractor. In this context, Kapitaniak obtained some interesting results for white noise using the Fokker–Planck equation [29,30]. He analyzed a Duffing oscillator under a stochastic distribution. An interesting outcome was the observation of a multi-modal probability distribution similar to nonlinear systems with chaotic properties.

We investigate different types of noise, namely, white, red, and green noise, because the sum of the power spectra for red and green noise is equal to the power spectrum for white noise. NIS can also occur for a set of oscillators, but we restricted our study to just two time-delayed systems. Instead, we approach the issue of NIS and NES in time-delayed systems using colored noise, which has not been explored yet to the best of our knowledge. The effect of parameter mismatch for NIS is also discussed. We present a numerical integration scheme for time-delayed systems in the presence of white and colored noise; most previous schemes for numerical integration are for non-delayed systems. Experimental observation of NIS in delay system is presented using electronic circuits. The main novelty of our study is experimental evidence of NIS for colored noise in time-delayed systems, whereas previous research has mostly been theoretical or numerical.

The remainder of the paper is organized as follows. The theory of NIS between two identical time-delayed systems and the basic properties of white and colored noise are discussed in Section 2. An expression for the maximum transverse Lyapunov exponent (MTLE) for NIS is presented and used to characterize NIS. Numerical simulations of NIS in the presence of different types of noise using Ikeda [31] and Mackey–Glass [32] systems are described in Section 3. NES simulations are presented in Section 4. In Section 5, the effects of parameter mismatch on NIS are discussed. Experimental observations of NIS using electronic circuits as time-delayed systems are described in Section 6. Section 7 concludes.

2. Theory of NIS

The general form of a noise-driven delayed dynamic system can be written as

$$\dot{x} = -ax + bf_1(x_\tau) + \xi \tag{1a}$$

$$\dot{y} = -ay + bf_2(y_\tau) + \xi, \tag{1b}$$

where a and b are positive constants, $x_\tau = x(t-\tau)$ and τ are the intrinsic system delays, $\xi(t)$ is the common noise, and $f_1(x_\tau)$ and $f_2(y_\tau)$ are nonlinear functions of x_τ and y_τ respectively. $f(x_\tau)$ characterizes the system; for example, $f(x) = -\sin x$ for an Ikeda model [31] and $f(x) = \frac{x}{1+x^{10}}$ for a Mackey–Glass system [32]. Systems (1a) and (1b) are driven by common noise without any interaction between them. Fig. 1 shows a diagram of NIS. In our simulation, we assume that the two systems are either identical ($f = f_1 = f_2$) or they have a slight parameter mismatch between (1a) and (1b). The noise driving the two systems is either white noise or colored noise. For colored noise, we considered red and green noise.

If $\xi(t) = w(t)$ represents Gaussian white noise, then it has the properties $\langle w(t)\rangle = 0$ and $\langle w(t)w(t')\rangle = 2D_w\delta(t-t')$, where $D_w > 0$ is the noise intensity, δ is Dirac's delta function, and $\langle\cdots\rangle$ denotes averaging over the realizations of $w(t)$.

We also consider the effect of red and green noise on chaos synchronization. For red noise, we replace $\xi(t)$ in Eq. (1) by $r(t)$, which can be generated by the Langevin equation [33]. The dynamic evolution of $r(t)$ is given by

$$\dot{r} = -\alpha r + \alpha w, \tag{2}$$