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Nonextensive nuclear liquid–gas phase transition

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HIGHLIGHTS

- We study the nuclear liquid–gas phase transition in the framework of nonextensive statistical mechanics.
- In asymmetric nuclear matter, mechanical and chemical thermodynamic instabilities are present.
- The liquid–gas phase transition results sensibly modified for small deviations from the standard Boltzmann–Gibbs statistics.
- Isospin fractionation effect turns out to be modified in presence of nonextensive statistics.

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ABSTRACT

We study an effective relativistic mean-field model of nuclear matter with arbitrary proton fraction at finite temperature in the framework of nonextensive statistical mechanics, characterized by power-law quantum distributions. We investigate the presence of thermodynamic instability in a warm and asymmetric nuclear medium and study the consequent nuclear liquid–gas phase transition by requiring the Gibbs conditions on the global conservation of baryon number and electric charge fraction. We show that nonextensive statistical effects play a crucial role in the equation of state and in the formation of mixed phase also for small deviations from the standard Boltzmann–Gibbs statistics.

1. Introduction

The study of the thermodynamic properties of strongly interacting nuclear matter and the related phase transitions under extreme conditions is one of the most important goals of heavy ion collision experiments at intermediate and high energies. At low temperatures ($T \leq 20$ MeV) and subnuclear densities, a liquid–gas type of phase transition was first predicted theoretically [1] and later observed experimentally in a nuclear multifragmentation phenomenon at intermediate-energy nuclear reactions [2,3].

Because nuclei are made of neutrons and protons, the nuclear liquid–gas phase transition is in a binary system where one has to deal with two independent proton and neutron chemical potentials for baryon number and electric charge conservation. In fact, the information coming from experiments with heavy ions in intermediate- and high-energy collisions is that the Equation of State (EOS) depends on the energy beam but also sensibly on the proton fraction Z/A (or isospin density) of the colliding nuclei [4]. Moreover, the study of nuclear matter with arbitrary electric charge fraction turns out to be important in radioactive beam experiments and in the physics of compact stars. Taking into account this important property, a very detailed study by Müller and Serot [5] focused on the main thermodynamic properties of asymmetric nuclear matter in the framework of a relativistic mean field model. Examples of other two-component systems are binary alloys and liquid ^{3}He with spin-up and spin-down fluids.

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A relevant aspect of a system with two conserved charges in asymmetric nuclear matter, is that, at variance with the so-called Maxwell construction for one conserved charge, the pressure is not constant in the mixed phase and therefore the incompressibility does not vanish [5,6]. Another interesting aspect of two-component systems is the possibility of having different proton–neutron ratios in the liquid and gas phases because of the symmetry energy, while still conserving the overall initial proton fraction (or isospin density). Moreover, for a binary system with two phases, the binodal coexistence surface is two dimensional and the instabilities in the mixed liquid–gas phase arise from fluctuations in the baryon density (mechanical instability) and in the proton concentration (chemical instability) [5,7,8]. Such a feature plays also a crucial role in the structure and in the possible hadron–quark phase transition in compact star objects [9,10].

Recently, there has been increasing evidence that the nonextensive statistical mechanics, proposed by Tsallis, can be considered as an appropriate basis to deal with physical phenomena where strong dynamical correlations, long-range interactions and microscopic memory effects take place [11–15]. A considerable variety of physical applications involve a quantitative agreement between experimental data and theoretical models based on Tsallis thermostatistics. In particular, in the last years there has been a growing interest in high energy physics applications of nonextensive statistics and several authors have outlined the possibility that experimental observations in relativistic heavy ion collisions can reflect nonextensive statistical behaviors [16–24].

Nonextensive statistical mechanics introduced by Tsallis consists of a generalization of the common Boltzmann–Gibbs statistical mechanics and it is based upon the introduction of entropy [13,25]

$$S_q[f] = \frac{1}{q-1}\left(1 - \int [f(\mathbf{x})]^q \, \mathrm{d}\Omega\right), \qquad \left(\int f(\mathbf{x})\, \mathrm{d}\Omega = 1\right), \tag{1}$$

where $f(\mathbf{x})$ stands for a normalized probability distribution, $\mathbf{x}$ and $\mathrm{d}\Omega$ denoting, respectively, a generic point and the volume element in the corresponding phase space (here and in the following we set the Boltzmann and the Planck constant equal to unity). The real parameter q determines the degree of non-additivity exhibited by the entropy form (1) which reduces to the standard Boltzmann–Gibbs entropy in the limit $q \to 1$. By means of maximizing the entropy S_q, under appropriate constraints, it is possible to obtain a probability distribution (or particle distribution) which generalized, in the classical limit, the Maxwell–Boltzmann distribution. The nonextensive classical distribution can be seen as a superposition of the Boltzmann one with different temperatures which has a mean value corresponding to the temperature appearing in the Tsallis distribution [19,23].

The existence of nonextensive statistical effects strongly affects the finite temperature and density nuclear EOS [26–29]. In fact, by varying temperature and density, the EOS reflects in terms of the macroscopic thermodynamical variables the microscopic interactions of the different phases of nuclear matter. The extraction of information about the EOS at different densities and temperatures by means of heavy ion collisions is a very difficult task and can be realized only indirectly by comparing the experimental data with different theoretical models, such as, for example, fluid-dynamical models. Related to this aspect, it is relevant to observe that a relativistic kinetic nonextensive theory [30] and a nonextensive version of a hydrodynamic model for multiparticle production processes have been proposed [31].

In this paper we are going to study the influence of nonextensive statistical effects on the thermodynamical instabilities in warm and asymmetric nuclear matter and we investigate how the phase diagram of the nuclear liquid–gas phase transition can be modified in the framework of nonextensive statistical mechanics.

2. Nonextensive hadronic equation of state

The basic idea of the relativistic mean field model, widely and successfully used for describing the properties of finite nuclei as well as hot and dense nuclear matter [32–34], is the interaction between baryons through the exchange of mesons. In the original version, we have an isoscalar–scalar σ meson field which produces the medium range attraction and the exchange of isoscalar–vector ω mesons responsible for the short range repulsion. The saturation density and binding energy per nucleon of nuclear matter can be fitted exactly in the simplest version of this model but other properties of nuclear matter, for example, incompressibility, cannot be well reproduced. To overcome these difficulties, the model has been modified introducing in the Lagrangian two terms of self-interaction for the σ mesons that are crucial to reproduce the empirical incompressibility of nuclear matter and the effective mass of nucleons M_N^*. Moreover, the introduction of an isovector–vector ρ meson allows to reproduce the correct value of the empirical symmetry energy.

In this framework, the Lagrangian density describing the nucleonic degrees of freedom (p, n) can be written as [35]

$$\begin{aligned}\mathcal{L}_N = {} & \bar{\psi}[i\gamma_\mu\partial^\mu - (M - g_\sigma\sigma) - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma^\mu\vec{\tau}\cdot\vec{\rho}_\mu]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ & - U(\sigma) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu},\end{aligned} \tag{2}$$

and $M = 939$ MeV is the vacuum nucleon mass. The field strength tensors for the vector mesons are given by the usual expressions $F_{\mu\nu} \equiv \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, and $U(\sigma)$ is a nonlinear potential of σ meson

$$U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4, \tag{3}$$

usually introduced to achieve a reasonable compression modulus for equilibrium nuclear matter.