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Effects of colored noise and noise delay on a calcium oscillation system

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HIGHLIGHTS

- Effects of colored noise and noise delay on a calcium oscillation system is investigated.
- The colored noise can induce coherence bi-resonance phenomenon in the system.
- There exist three peaks in the reciprocal coefficient of variance vs the self-correlation time curves.
- Large re-injected parameter reduces noise induced spikes regularity.

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ABSTRACT

As a calcium oscillations system is in steady state, the effects of colored noise and noise delay on the system is investigated using stochastic simulation methods. The results indicate that: (1) the colored noise can induce coherence bi-resonance phenomenon. (2) there exist three peaks in the R–τ_0 (R is the reciprocal coefficient of variance, and τ_0 is the self-correlation time of the colored noise) curves. For the same noise intensity $Q = 1$, the Gaussian colored noise can induce calcium spikes but the white noise cannot do this. (3) the delay time can improve noise induced spikes regularity as τ_0 is small, and R has a significant minimum with increasing τ as τ_0 is large. (4) large values of ζ reduce noise induced spikes regularity.

1. Introduction

Noise induced non-equilibrium phenomena have attracted a great deal of attention in the last two decades. One example is the phenomenon of stochastic resonance (SR). SR phenomenon was originally discovered by Benzi et al. [1] and Nicolis et al. [2], in which the periodic changes of the ancient climate were investigated. Since then, scientists have found the SR phenomenon in various fields, such as ring laser system [3], optical bistable system [4], chemical reactions system [5,6], neuron system [7–9], and biological system [10–14]. The achievements about SR have been comprehensively summarized by Gammaitoni et al. and Wellens et al. [15,16]. With the development of SR studies, it has been shown that without an external periodic signal coherence of noise-induced oscillations is maximized for an optimal level of noise intensity. This phenomenon was termed as coherence resonance (CR) [17,18]. In recent years, some interest has been devoted to the studies of CR phenomenons in nonlinear systems [19–21]. Lee et al. have investigated the nonlinear response of the Hodgkin–Huxley model without external periodic signal to the noisy synaptic current near the saddle–node bifurcation of limit cycles, and found the coherence resonance phenomena occurs [19].

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In many natural and physical situations, the time delay is usually used to describe an intrinsic delay mechanism or an introduction of the time-delayed feedback, which implies that the dissipative evolution depends on the state of the system in a shifted previous time. Thus, the effects of time delay on stochastic system have received a lot of attention; such as clustering [22], multi-stability [23], amplitude death [24,25], anticipated synchronization [26,27] and so on. Besides the time-delayed feedback through the system variable, other techniques such as noise delay [28–30] have also been proposed to control the dynamics of stochastic systems. In 2007, Borromeo et al. studied the non-equilibrium escape dynamics in a bistable system under the influence of primary noise source and a recycling term, and found that under stationary conditions, an appreciable fraction of renewal trajectories gets locked to the noise-recycling delay time [31].

Many important cellular functions are regulated by intra- and intercellular Ca^{2+} signals. Ca^{2+} is a primary regulator in many intracellular physiological processes such as the early response to injury of brain tissue, neurotransmitter release, synaptic plasticity and so on. It mediates the subsequent activity of cells and is invariably involved in cell death. It is shown that the information is mainly encoded in the frequency of calcium signals. In Ca^{2+} oscillations system, various fluctuations and time delay cannot be negligible, and complex intracellular Ca^{2+} signals in the presence of noise have been investigated experimentally and numerically [32–39]. It has been suggested that the calcium oscillations system may use stochastic resonance dynamics to improve its signal periodicity or coherence [33,35].

In the present article, we investigated the influence of colored noise and noise delay term on a calcium oscillations system, i.e., the system is subject to a delay correlated noise generated by the superposition of a primary Gaussian colored noise source and a secondary component, re-injected after delay time τ. In Section 2, the deterministic model of the system is introduced and then the stochastic model driven by colored noise and noise delay term is derived. In Section 3, the effects of noise and noise delay on the system were investigated. In Section 4, we get the conclusions.

2. The calcium oscillations system with Gaussian colored noise and noise delay

There are a number of Ca^{2+} oscillations system models, all of which show a great deal of fundamental similarity. The model used in the present article was proposed by Höfer [40,41] and Gracheva [42]. If the noise is ignored, the time evolution of the species is governed by the following macroscopic kinetics

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \rho\left(\nu_0 + \nu_c\frac{p}{k_0+p} - \nu_4\frac{x^2}{k_4^2+x^2}\right) + \rho\left(\frac{\alpha k_r(x,p)}{\beta}(z-(1+\beta)x) - \alpha\nu_3\frac{x^2}{k_3^2+x^2}\right), \tag{1}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \rho\left(\nu_0 + \nu_c\frac{p}{k_0+p} - \nu_4\frac{x^2}{k_4^2+x^2}\right), \tag{2}$$

$$k_r(x,p) = k_1\left(\frac{d_2(d_1+p)px}{(d_p+p)(d_a+x)(d_2(d_1+p)+x(d_3+p))}\right)^3 + k_2$$

x and z represent the concentration of the cytosolic Ca^{2+} and the total Ca^{2+} in the cell, respectively. The structural parameters $\alpha = 2.0\ \mathrm{L}\cdot\mu\mathrm{mol}^{-1}$, $\beta = 0.1\ \mathrm{L}\cdot\mu\mathrm{mol}^{-1}$, $\rho = 0.2\ \mathrm{L}\cdot\mu\mathrm{mol}^{-1}$. $\nu_0 = 0.2\ \mu\mathrm{mol}^{-1}\cdot\mathrm{L}^{-1}\cdot\mathrm{s}^{-1}$ describes a calcium leakage from the background. $\nu_c = 4.0\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}\cdot\mathrm{s}^{-1}$ is the maximum rate of IP_3 (inositol 1,45-trisphosphate) induced calcium formation (influx), $\nu_3 = 9.0\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}\cdot\mathrm{s}^{-1}$ is the maximum rate of ER (endoplasmic reticulum) uptake of calcium from the cytosol, $\nu_4 = 3.6\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}\cdot\mathrm{s}^{-1}$ is the maximum rate of calcium extrusion through the plasma membrane. Other parameters $k_0 = 4.0\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $k_2 = 0.02\ \mathrm{s}^{-1}$, $k_3 = 0.12\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $k_4 = 0.12\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $d_1 = 0.3\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $d_2 = 0.4\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $d_3 = 0.2\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $d_p = 0.2\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$, $d_a = 0.4\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$.

$k_r(x,p)$ is the IP_3R (IP_3 receptor) release function and describes the gating kinetics of the IP_3 receptor. p is the concentration of IP_3. When p is viewed as the control parameter, the bifurcation diagram is shown in Fig. 1, and the two bifurcation points appear at $p = 1.450\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$ and $p = 8.892\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$. In the absence of noise part, the system is in steady state at $p_0 = 1.40\ \mu\mathrm{mol}\cdot\mathrm{L}^{-1}$. To study the effects of fluctuations on this system, we apply Gaussian colored noise and noise delay to p as $p = p_0 + \xi(t)$, $\xi(t)$ is the noise and noise delay term,

$$\xi(t) = \eta(t) + \zeta * \eta(t-\tau), \tag{3}$$

$|\zeta| \leq 1$ is the parameter of noise re-injected into the system after delay time τ. In this article, we only consider $0 \leq \zeta \leq 1$. $\eta(t)$ is the Gaussian colored noise in the following forms

$$\langle\eta(t)\rangle = 0, \tag{4}$$

$$\langle\eta(t)\eta(t')\rangle = \frac{Q}{\tau_0}\exp\left(-\frac{|t-t'|}{\tau_0}\right). \tag{5}$$

Q and τ_0 are the noise intensity and self-correlation time, respectively. We can introduce another equation as follows [43]

$$\frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = -\frac{\eta(t)}{\tau_0} + \frac{\Gamma(t)}{\tau_0}, \tag{6}$$

$\Gamma(t)$ is Gaussian white noise with $\langle\Gamma(t)\rangle = 0$ and $\langle\Gamma(t)\Gamma(t')\rangle = 2Q\delta(t-t')$.