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Effects of time delay on transport processes in an active Brownian particle☆

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HIGHLIGHTS

- The transport of an active Brownian particle with time delay is investigated.
- Analytical expressions for the mean velocity and effective diffusion coefficient are derived.
- There is a value in the bias, below or above which the delay, enhances or weakens the diffusion.
- Our investigation may be helpful to understand the self-propelled motion of biological processes.

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ABSTRACT

The transport properties of an active Brownian particle with a time-delayed feedback and an external bias are investigated theoretically. By virtue of the perturbation theory for small delay, analytical expressions for the mean velocity and effective diffusion coefficient are derived. There exists a critical absolute value of the bias, below and above which the delay, respectively, enhances and weakens the diffusion, for a fixed noise intensity. The effects of delay observed above are more pronounced for weaker noise. These results are further verified via direct numerical simulations.

1. Introduction

Transport of Brownian particles continues to attract enormous interest due to the possible applications in many different contexts of physics, chemistry, and biology [1–3]. One of the surprising phenomena, termed enhancement of diffusion, has stimulated extensive research [4–8]. Generally speaking, this phenomenon is characterized by the existence of a critical parameter value, around which the diffusion coefficient presents a pronounced peak. This behavior has also been demonstrated experimentally in tracking the motion of colloidal spheres [9]. In order to describe self-propelled motion recently, in biology ranging from flocks of animals to single cell motility, one class of models – active Brownian particles (ABPs) [10] – has been scrutinized in a huge number of works [3,11–15]. ABPs can transform the internal energy into energy of motion that drives them out of equilibrium. Essential for this behavior is not the mere speed dependence of the friction coefficient, but also that the friction coefficient is negative over a range of velocities: the positive and negative friction coefficients are corresponding to the dissipation and the uptake of energy, respectively [12–15]. In context of ABPs, the enhancement of diffusion has been demonstrated in the weak noise limit [13].

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From the point of physics, the transport of matter, energy and information through the system requires finite time (ascribed to a finite transmission speed), which is usually treated as time delay. The delayed feedback effect has been intensively studied, e.g., in biological systems as models to describe the tumor–immune interactions [16–18], visual feedback [19], and brain activity [20], to mention but a few. Recently, it also has been exploited as a viable tool to manipulate and optimize transport properties [21–27]. To the best of our knowledge, the feedback effects on the transport processes of ABPs has received comparably little attention [3,10–15]. Evidently, as mentioned above, it is interesting to examine this issue. In our present study, we consider a time-delayed feedback in a model put forward by Lindner and Nicola [13], which is established as a model of the self-propelled motion. We analytically derive the giant diffusion and reliable directed transport curves with varying delay time, for a fixed noise intensity. From the experimental viewpoint [28], it is accessible that the enhancement of diffusion occurs, specifically, within some finite temperature interval (as opposed to the weak noise limit) via appropriately designing the working parameters. An inclusion of time-delayed feedback here provides an alternative scenario to achieve the diffusion enhancement.

In Section 2, an ABP model with time delay is introduced first and the theoretical analysis is derived. Then, the effects of delay on its transport processes are studied both analytically and numerically, and the related discussions are conducted. In Section 3, a brief conclusion ends the paper.

2. Transport properties of an ABP with a bias and time-delayed feedback

The time evolution of an ABP considered in this paper are described by the stochastic delay differential equation:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v, \tag{1}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = v_\tau - v^3 + f + \xi(t), \tag{2}$$

where x and v are the displacement and velocity of the particle at time t, respectively; $v_\tau = v(t-\tau)$ and τ is delay time; f is a bias; $\xi(t)$ is Gaussian white noise with $\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t')$, where D is the noise intensity. An ABP obeying Eqs. (1) and (2) displays a mean velocity $\langle v\rangle = \lim_{t\to\infty}\langle x(t) - x(0)\rangle/t$, and a diffusion, characterized by the effective diffusion coefficient spread around $\langle v\rangle$, $D_{eff} = \lim_{t\to\infty}[\langle x(t)^2\rangle - \langle x(t)\rangle^2]/(2t)$ [29]. In the absence of delay ($\tau = 0$), the velocity-dependent potential $U(v) = v^4/4 - v^2/2 - fv$ has two steady states $v_\pm$ and one unstable steady state v_u for the small bias $|f| \le \frac{2\sqrt{3}}{9}$. In this case ($\tau = 0$), the critical force that separates parameter regimes of giant diffusion from the regimes with reliable directed transport was investigated by Lindner and Nicola [13], and the transport properties of the corresponding coupled systems were studied by Wu and Zhu [30,31].

Here, we would like to focus on the effects of delay on the transport of the system. It should be noted that the ansatz for the time-delayed feedback in Eq. (2) is *linear, the simplest and only of heuristic form. We would like to stress that our work does not intend to model any particular biological or social object but instead to analyze a physical system with time-delayed feedback exhibiting new types of dynamics. The nonlinear feedback (an analytical treatment is in general rather difficult), which, for instance, facilitates the appearance of spike death* [32], *is not explored in the present work. The transport properties, generally speaking, should depend on the particular form of delayed feedback. It remains here to be seen which form of feedback is more accurate in real biological systems. A more interesting possibility is that various forms of time-delayed feedback may respond to different realistic systems for transferring matter, energy and information* [33]. *We expect that the findings in this work will trigger more investigations on time-delayed ABPs to address the issue in depth.*

To begin with, the *Fokker–Planck* equation corresponding to Eq. (2) can be obtained [34]

$$\frac{\partial P(v,t)}{\partial t} = -\frac{\partial}{\partial v}\int (v_\tau - v^3 + f)P(v,t;v_\tau,t-\tau)\mathrm{d}v_\tau + D\frac{\partial}{\partial v}\int P(v,t;v_\tau,t-\tau)\mathrm{d}v_\tau, \tag{3}$$

with $P(v,t;v_\tau,t-\tau)$ the conditional probability density. As the delay time is small ($\tau \le 1$), using the perturbation theory the steady probability distribution (SPD) in the first-order approximation can be solved analytically [34]

$$P_{st}(v) = N\exp\left(-\frac{U_{eff}(v)}{D}\right), \tag{4}$$

with

$$N = \frac{1}{\int_{-\infty}^{\infty}\exp\left(-\frac{U_{eff}(v)}{D}\right)\mathrm{d}v}, \tag{5}$$

$$U_{eff}(v) = (1+\tau)\left(\frac{v^4}{4} - \frac{v^2}{2} - fv\right). \tag{6}$$

From the expressions of effective potential U_{eff} and U, $U_{eff}(v) = (1+\tau)U(v)$ is clearly apparent and thus the positions of $v_\pm$ and v_u do not change in the presence of delay. The U_{eff}, without loss of generality, for different delay times are plotted in