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# Dynamical properties for a mixed Fermi accelerator model

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## HIGHLIGHTS

- Decay of energy in a dissipative system.
- Corridors created by stable manifolds.
- Theoretical prediction of the energy decay.

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ABSTRACT

The behavior of the decay of velocity in a semi-dissipative one-dimensional Fermi accelerator model is considered. Two different kinds of dissipative forces were considered: (i) $F \propto -v$ and; (ii) $F \propto -v^2$. We prove the decay of velocity is linear for (i) and exponential for (ii). During the decay, the particles move along specific *corridors* which are constructed by the borders of the stable manifolds of saddle points. These corridors organize themselves in a very complicated way in the phase space leading the basin of attraction of the sinks to be seemingly of fractal type.



## 1. Introduction

The one-dimensional Fermi accelerator model was firstly studied by Enrico Fermi who proposed a mechanism to possible explain the acceleration of cosmic rays in the interstellar medium [1]. He assumed that the cosmic particles were accelerated by moving magnetic clouds present in the cosmos. A derivation of this model for a non-relativistic classical particle (representing the cosmic ray) bouncing between walls was then proposed by Ulam [2]. In the model, one wall is moving smoothly and periodically in time, therefore making allusions to the moving magnetic fields, while the other one is considered to be fixed, working as a returning mechanism for the particle to experience a further collision with the moving wall. The model then is known as the Fermi–Ulam model [3–5] and has been studied in many different versions and along several approaches [6–15]. The dynamics of the particle is generally described by a two-dimensional non-linear mapping for the velocity of the particle and time. For periodic oscillations and in the absence of dissipation, the phase space presents a mixed structure in the sense that periodic islands are surrounded by a chaotic sea which is limited by invariant spanning curves. When dissipation is introduced, generally the elliptic islands turn into sinks via a bifurcation. The spanning curves are destroyed and attractors are observed. The model is often used to relate problems involving unlimited energy growth of the bouncing particle, a phenomenon also called Fermi acceleration (FA). Applications of FA have been observed in different

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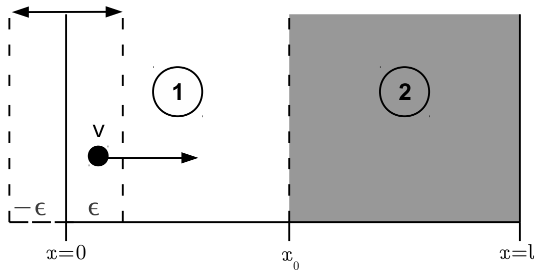


**Fig. 1.** Sketch of the mixed Fermi accelerator model. Regions 1 and 2 have a different viscous drag force. The equation of the moving wall is given by $x_p(t_n) = \varepsilon\cos(\omega t_n)$.

areas of science including plasma physics [16], astrophysics [17,18], atomic physics [19], optics [20,21] and even in time dependent billiard problems [22]. The phenomenon however seems not to be robust since dissipation is assumed to be a mechanism to suppress Fermi acceleration [23]. The dynamics of particles confined in walls can be used also to describe properties of the so called cavity Optomechanics [24,25], mechanical [26,27] and nano-mechanical resonators [28]. Indeed in such systems, as a influence of the injection of a laser beam, transference of momentum to a mirror (wall) is observed therefore leading to a kind of synchronization of micromechanical [29] or nano-mechanical resonators [30].

In real experiments, dissipation is always present. Therefore one way to consider the presence of damping in the system is to assume that collisions of the particle with the walls are inelastic. It then leads the particle to experience a fractional loss of energy at each collision. The system does not preserve the phase space measure as observed in the non-dissipative case and the mixed structure is changed. In particular, it is possible to observe different asymptotic behavior as the damping coefficient is varied. Among them, effects of transient [31], attractive fixed points [32], chaotic attractors [33], and the occurrence of boundary crisis [34] can all be considered. A different way of introducing dissipation in the system is to consider that the particle is moving along the presence of a viscous drag force, like a fluid. In this paper we consider the effects of a dissipative force of two types [35]: (i) $F \propto -v$ and; (ii) $F \propto -v^2$ where $v$ is the velocity of the particle. The $(-)$ in the expression denotes the force is contrary to the movement of the particle. However, the fluid is not present in the entire accessible region. The distribution of the gas in the system is controlled by a control parameter $\lambda$ in the sense that for $\lambda = 1$ the system is non-dissipative and for $0 \le \lambda < 1$ there is a presence of gas in part of the system. Our main goal is to study and characterize the behavior of the velocity of the particle as a function of the number of collisions with the moving wall as well as the control parameters for cases (i) and (ii). As we will show, when the initial velocity is sufficiently high, the particle experiences a decay of velocity which we prove to be: linear for case (i) and exponential for case (ii). Numerical results remarkably give support to our analytical approach. During the decay, we shown that the particle moves in the phase space along specific region we denote as *corridors* whose borders are generated by stable manifolds leaving from a saddle point. These corridors, corresponding to the region delimited by the borders of the stable manifolds, are indeed the basin of attraction of the attractors, mainly the sinks. So far we can tell that this might be the first time that manifolds have been used to defined the corridors' path to describe the decay of energy in such type of systems. Applications to higher dimensional systems are expected to be valid too.

The organization of the paper is as follows. In Section 2 we describe the model and the equations that describe the dynamics for case (i), i.e. when $F \propto -v$. The analytical approach for a decaying particle is constructed and numerical simulations are presented in support of the analytical approach. Section 3 is devoted to discussing the case (ii), $F \propto -v^2$. The decay of velocity is proved to be of exponential type and confirmed by numerical simulations. Our concluding remarks are presented in Section 4.

## 2. The model, the mapping and results for the case $F \propto -v$

The mixed Fermi accelerator model we are considering here consists of a classical particle of mass $m$ which is confined to bounce between two rigid walls. One of them is fixed and the other one moves periodically in time. The collisions of the particle with either walls are assumed to be elastic. The fixed wall is located at $x = l$ and the moving wall has the equation of the position given by $x_w = \varepsilon\cos(\omega t)$. Here $\varepsilon$ is the amplitude of oscillation of the moving wall, $\omega$ is the angular frequency and $t$ is time. The region within the interval $x \in [-\varepsilon, \varepsilon]$ is called the collision zone. The velocity of the moving wall is given by $v_w = \mathrm{d}x_w/\mathrm{d}t = -\varepsilon\omega\sin(\omega t)$. We consider the existence of two different regions between the walls, as shown in Fig. 1. Region 1 is given defined by the space between the oscillating wall $x_w$ and the point $x_0$ while region 2 is given by $x_0 < x \le l$. It is convenient to introduce the parameter $\lambda = x_0/l$. In the model, we assume that region 1 has no dissipative forces while region 2 has a force like $F = -\eta' v$. The case of $F = -\eta' v^2$ will be considered in Section 3.

As usual in the literature, the dynamics of the model is described by a two dimensional non-linear mapping for the variables $(v, t)$, where $v$ and $t$ are respectively the particle's velocity after the collision with the moving wall and the time at the collision. When moving in the viscous region, the particle is experiencing continuously a reduction of its velocity. The mapping $T$ is given by $T(v_n, t_n) = (v_{n+1}, t_{n+1})$ where the index denotes respectively collisions $n$th and $(n+1)$th. To construct the mapping and to avoid privileging any initial condition, we assume that at the instant $t = t_n > 0$, the particle is in the position $x_p(t_n) = \varepsilon\cos(\omega t_n)$ with velocity $v = v_n$. Such a choice is made because it corresponds indeed to the