Physica A 392 (2013) 4271–4278

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Lattice bosons in a quasi-disordered environment

R. Ramakumar [a,*], A.N. Das [b]

[a] Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India

[b] Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700064, India

HIGHLIGHTS

- We study the harmonically confined 1D lattice bosons with Aubry–André disorder.
- The disorder reduces the condensate fraction appreciably.
- Low-T depletion rate of the condensate increases with increasing disorder strength.

ARTICLE INFO

Article history:
Received 12 December 2012
Received in revised form 4 May 2013
Available online 27 May 2013

Keywords:
Bose–Einstein condensation
Optical lattices
Disorder effects
Thermodynamic properties

ABSTRACT

In this paper, we study non-interacting bosons in a quasi-disordered one-dimensional optical lattice in a harmonic potential. We consider the case of deterministic quasi-disorder produced by an Aubry–André potential. Using exact diagonalization, we investigate both the zero temperature and the finite temperature properties. We investigate the localization properties by using an entanglement measure. We find that the extreme sensitivity of the localization properties to the number of lattice sites in finite size closed chains disappear in open chains. This feature continues to be present in the presence of a harmonic confining potential. The quasi-disorder is found to strongly reduce the Bose–Einstein condensation temperature and the condensate fraction in open chains. The low temperature thermal depletion rate of the condensate fraction increases considerably with increasing quasi-disorder strength. We also find that the critical quasi-disorder strength required for localization increases with increasing strength of the harmonic potential. Further, we find that the low temperature condensate fraction undergoes a sharp drop to 0.5 in the localization transition region. The temperature dependence of the specific heat is found to be only marginally affected by the quasi-disorder.

1. Introduction

The prediction of the localization of Bloch waves moving through a randomly disordered crystal is one of the fundamental results in quantum mechanics [1]. Anderson localization of matter waves is a strong function of the system dimensionality. In two and lower dimensions, all the single particle states are localized by non-zero disorder [2,3]. Anderson localization of matter waves was directly observed in several experiments recently [4–7]. The localization of all the single particles states in one and two dimensions occurs for random disorder (see also the note in Ref. [8]). However, if the disorder distribution is deterministic, localization occurs only beyond a critical disorder strength. A particularly simple model in which such a transition from the extended to the localized states occurs is the one-dimensional Aubry–André (AA) model [9]. Among the experiments mentioned above, an experiment of particular interest for the purpose of this paper are the studies [5], by the LENS group, of the localization of a Bose–Einstein condensate of non-interacting lattice bosons subjected to the Aubry–André

* Corresponding author. Tel.: +91 9818156649.
E-mail addresses: rkumar@physics.du.ac.in, rramak2009@gmail.com (R. Ramakumar).

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http://dx.doi.org/10.1016/j.physa.2013.05.018

potential. Our primary motivation comes from these studies in which a finite one-dimensional (1d) Aubry–André model was experimentally realized. There have been extensive theoretical studies of the AA model in the past [10–17], and in the recent years [18–27] in response to the experimental developments mentioned above (for recent reviews see Refs. [28–30]).

In this paper, our main focus is the finite temperature properties of bosons in finite 1d optical lattices with AA quasi-disorder in harmonic confining potentials. Before going to the studies of a Bose condensate in the AA potential in a harmonic trap, we look at the sensitivity of localization properties of a single boson in finite optical lattices with open and closed boundary conditions. We note that the open boundary conditions are more appropriate for the experiments involving bosons in optical lattices. We will show that the extreme sensitivity of the localization properties of a particle in the AA model with closed boundary conditions disappear when open boundary conditions are used. This feature is important when we consider the many-boson system in finite lattices since now one may use any lattice size. Though this continues to be the overall feature after the application of an additional harmonic confining potential, we find a small delocalization tendency before the complete localization transition occurs. This feature results from a competition between the influences of the harmonic potential and the quasi-disorder potential. Further, we go on to study Bose condensation in the AA potential. We study the effects of increasing quasi-disorder on the condensation temperature, condensate fraction, and the specific heat of the system. The quasi-disorder is found to have significant effects on some of these properties. The quasi-disorder strongly reduces the condensation temperature and the condensate fraction near the localization transition. We also find that the fall in the condensate fraction shifts to larger quasi-disorder strengths with increasing harmonic potential strength. Since the focus of this paper are the effects of deterministic quasi-disorder on lattice bosons in harmonic potentials, it is fitting for us to place these studies in the general context of existing work on bosons in disordered environments. In the past, the effects of random disorder on interacting 3d continuum bosons have been extensively investigated [31–39]. It has been found that both the condensation temperature and the condensate fraction decrease with increasing disorder strength. In the case of non-interacting lattice bosons, the condensation temperature was found [40,41] to decrease for a small filling while showing the opposite trend for a large filling. There have been extensive studies [42–47] on the effects of disorder on interacting lattice bosons within the framework of the disordered Bose–Hubbard model. Recent experimental studies [48] of interacting disordered lattice bosons in a harmonic trap find a decreasing condensate fraction with increasing disorder strength.

The rest of this paper is organized as follows. The study of the single particle localization properties in the open and the closed finite length chains is given in the next section (Section 2). The study on the influence of the quasi-disorder strength on the condensation temperature, condensate fraction, and the specific heat is presented in Section 3. The conclusions are given in Section 4.

2. The effects of boundary conditions on localization

As stated, we consider harmonically confined lattice bosons with AA disorder potential. The Hamiltonian of this system is:

$$H = -t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \left[Ka^2 i^2 + \lambda \mathrm{Cos}(2\pi q i) - \mu \right] c_i^\dagger c_i, \tag{1}$$

where t is the kinetic energy gain when a boson hop from site i to its nearest neighbor site j in a one-dimensional optical lattice with a lattice constant a, $c_i^\dagger$ is a boson creation operator, K the strength of the harmonic potential, λ the strength of the AA potential, and $q = (\sqrt{5} + 1)/2$ is the incommensurability parameter. Here t, Ka^2, and λ have energy units. All the energies in this paper are measured in units of t. After writing this Hamiltonian in a single particle site basis ($|i\rangle$), we numerically diagonalize it to obtain its eigen-energies and eigen-functions for various lattice sizes, quasi-disorder strengths, and the harmonic potential strengths. The localization properties of the eigen-functions are monitored by calculating the Shannon entropy which measures the quantum entanglement [49].

The Shannon entropy is given by

$$S = - \sum_i p_i \log_2 p_i, \tag{2}$$

where,

$$p_i = |a_i|^2 \tag{3}$$

in which the a_i is the ith site amplitude of the ground state wave function

$$|\psi_G\rangle = \sum_i a_i |i\rangle. \tag{4}$$

The quantum entanglement is maximum for a fully extended state and is zero for a state localized on a site.

The results of our calculations for open and closed chains [50], in the absence of the harmonic potential, are shown in Fig. 1. In the (finite size) closed chain results shown, the entanglement (S) falls abruptly at $\lambda \approx 2$ signifying a transition from an extended state to a localized state for lattices with 144 and 89 sites, but not for other numbers of lattice sites as shown in