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Self-organization of five species in a cyclic competition game[☆]

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HIGHLIGHTS

- We propose an extended "rock–paper–scissors" game model named the "fingers" game.
- We investigate the "fingers" game using both direct simulations and nonlinear partial differential equations.
- Increasing the number of species can jeopardize biodiversity.
- Reproduction rate and mobility also affect species' biodiversity in our game.

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ABSTRACT

Cyclic competition game models, particularly the "rock–paper–scissors" model, play important roles in exploring the problem of multi-species coexistence in spatially ecological systems. We propose an extended "rock–paper–scissors" game to model cyclic interactions among five species, and find that two of the five can coexistent when biodiversity disappears, which is different from the "rock–paper–scissors" game. As the number of fingers is five, we named the new model the "fingers" game, where the thumb, forefinger, middle finger, ring finger, and little finger cyclically dominate their subsequent species and are dominated by their former species. We investigate the "fingers" model in two ways: direct simulations and nonlinear partial differential equations. An important finding is that the number of species in a cyclic competition game has an influence on the emergence of biodiversity. To be specific, the "rock–paper–scissors" model is in favor of maintaining biodiversity in comparison with the "fingers" model when the variables (population size, reproduction rate, selection rate, and migration rate) are the same. It is also shown that the mobility and reproduction rate can promote or jeopardize biodiversity.

1. Introduction

Biodiversity and self-organized patterns are two fundamental phenomena of ecological systems. Over the last few years, there has been an increasing interest in understanding the dynamical mechanism of creating biodiversity and self-organized patterns in ecological systems. Generally, cyclic competition game models, particularly the "rock–paper–scissors" game model, are proposed to characterize the essence of multi-species ecological systems [1–5]. In fact, there are series of natural cyclic interactions in ecological systems which can be appropriately modeled as cyclic competition games, such as the three-morph mating system in the side-blotched lizard [6] and Escherichia coli [7].

Studies of cyclic competition games can greatly benefit our understanding of multi-species ecological systems. On the one hand, such a study can be used to explore how biodiversity is maintained or jeopardized [8–11,7,12–19]. For example, Kerr et al. [7] used a real-life "rock–paper–scissors" game to study the effect of local dispersal on biodiversity.

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Indeed, many insights have been gained about the creation of biodiversity through this approach. It was found that mobility could remarkably promote and jeopardize biodiversity [12–15]. In detail, when the mobility exceeded a threshold, biodiversity was destroyed; in contrast, when the mobility was below a certain threshold, three species could coexist. Moreover, it was found that competition and intraspecies infection could strongly promote coexistence and maintain ecological biodiversity, while interspecies spreading could not [16,17]. Jiang et al. [19] observed that increasing the selection rate could promote biodiversity.

On the other hand, studies of cyclic competition games can also be used to investigate the formation of self-organized patterns [20–26]. For example, Wang et al. [24] incorporated both intra- and inter-patch migrations in cyclic competition games, and found the occurrence of remarkable target-wave patterns in the absence of any external control. Jiang et al. [25] incorporated a periodic current of three species in a small central area to investigate the emergence of target waves. They also reproduced multi-armed spirals and multi-pair antispirals by using a set of seed species distributions.

Though much work has been done, most of them focus on the dynamics of cyclic competition games with three species, i.e., the "rock–paper–scissors" game. Indeed, until now, the dynamical behavior of cyclic competition games with more species was still not clear. In the "predator–prey" model, an extended version with four species has been investigated, in which each species dominated its subsequent species and was dominated by its former species cyclically [27–31]. It is shown that simply adding to the number of species can produce much more complex scenarios. Naturally, we wonder what happens to the dynamics behaviors of the "rock–paper–scissors" game after adding to the number of species. Following this line, we establish an extended "rock–paper–scissors" model with five species, where each species dominates its subsequent species and is dominated by its former species cyclically, and explore its dynamical behaviors in this paper.

The main contributions of this paper are two-fold. First, we show that two species may coexist in a cyclic competition game with five species, if neither of those two species could dominate or be dominated by the other. As a comparison, note that in the "rock–paper–scissors" model, any two species could not coexist without the presence of the third species. Second, we show the effects of mobility M, reproduction rate, and the number of species on biodiversity in a cyclic competition game with five species. In detail, we find that there is a critical threshold M_c for mobility. For $M > M_c$, at least three species become extinct. For $M < M_c$, five species could coexist. Intriguingly, we observe that the critical threshold for mobility is smaller in the cyclic competition game with five species than in the "rock–paper–scissors" game. This indicates that the number of species in cyclic competition may have an influence on the emergence of biodiversity.

This paper is organized as follows. Section 2 introduces the evolutionary model of the cyclic competition game with five species. Section 3 presents the main results: the effects of mobility, reproduction rate, and number of species on biodiversity are explored in this section. In Section 4, we explain the main results analytically. Some concluding remarks are given in Section 5.

2. The model

In this section, we propose an extended "rock–paper–scissors" game to model the cyclic interactions among five mobile species. For simplicity, we name this extended "rock–paper–scissors" game the "fingers" game, where the thumb (A), forefinger (B), middle finger (C), ring finger (D), and little finger (E) refer to each species, respectively. In this "fingers" game, the five species form a cycle such that each species dominates its subsequent species and is dominated by its former species. Obviously, there exist certain pairs of species where one cannot dominate or be dominated by the other. We call such pairs of species irrelevant species.

Now consider a square lattice of size $N = L^2$ with periodic boundary conditions. The population is arranged on this square lattice. In detail, each site in the square lattice is either occupied by one individual or empty. Interactions occur among two nearest neighboring individuals, as illustrated in the following rules:

$$AB \xrightarrow{\sigma} A\varnothing, \quad BC \xrightarrow{\sigma} B\varnothing, \quad CD \xrightarrow{\sigma} C\varnothing, \quad DE \xrightarrow{\sigma} D\varnothing, \quad EA \xrightarrow{\sigma} E\varnothing \tag{1}$$

$$A\varnothing \xrightarrow{\mu} AA, \quad B\varnothing \xrightarrow{\mu} BB, \quad C\varnothing \xrightarrow{\mu} CC, \quad D\varnothing \xrightarrow{\mu} DD, \quad E\varnothing \xrightarrow{\mu} EE \tag{2}$$

$$A\square \xrightarrow{\varepsilon} \square A, \quad B\square \xrightarrow{\varepsilon} \square B, \quad C\square \xrightarrow{\varepsilon} \square C, \quad D\square \xrightarrow{\varepsilon} \square D, \quad E\square \xrightarrow{\varepsilon} \square E. \tag{3}$$

Here, A, B, C, D, and E denote individuals from the five species, respectively, $\varnothing$ denotes empty sites, and $\square$ represents a general site which may be occupied with an arbitrary species or an empty site. Relation (1) describes the interactions of cyclic selection. Each species dominates a less-predominant species cyclically and leaves the neighboring site empty with rate σ. Relations (2) and (3) characterize reproduction and migration that occur at rate μ and ε, respectively.

The evolutionary dynamics of the population is illustrated as follows. At each step, we randomly choose one individual and one of its neighbors. For each selected pair of nodes, selection, reproduction, and migration occur with probabilities $\frac{\sigma}{\sigma+\mu+\varepsilon}$, $\frac{\mu}{\sigma+\mu+\varepsilon}$, and $\frac{\varepsilon}{\sigma+\mu+\varepsilon}$, respectively. However, whether the updating can successfully occur is determined by the state of both sites. In simulations, an actual time step is defined as the steps during which each individual experience one updating on average. In other words, in one actual step, N pairwise interactions occur. Moreover, the number of time steps involved in the simulation is called the waiting time T. Following previous works [12–14], here, the waiting time is set as $T = O(N)$. Individual mobility M is defined as $M = \varepsilon/2N$, which is proportional to the average area explored by a mobile individual per unit time according to the theory of random walks [32].