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The random field Ising model with an asymmetric and anisotropic bimodal probability distribution

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ABSTRACT

The Ising model in the presence of a random field is investigated within the mean field approximation based on Landau expansion. The random field is drawn from the asymmetric and anisotropic bimodal probability distribution $P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + \lambda * h_0)$, where the site probabilities p, q take on a value within the interval [0, 1] with the constraint $p + q = 1$, h_i is the random field variable with strength h_0 and λ is the competition parameter, which is the ratio of the strength of the random magnetic field in the two directions $+z$ and $-z$; λ is considered to be positive, resulting in competing random fields. For small and large values of p ($p < \frac{13-\sqrt{13}}{26}$ or $p > \frac{13+\sqrt{13}}{26}$, respectively) the phase transitions are exclusively of second order, but for $\frac{13-\sqrt{13}}{26} \leq p \leq \frac{13+\sqrt{13}}{26}$ they are of second order for high temperatures and small random fields and of first order for small temperatures and high/small random fields irrespective of the λ-value; in the latter case the two branches are joined smoothly by a tricritical point confirming, in this way, the existence of such a point. In addition, re-entrant phenomena can be seen for appropriate ranges of the temperature and random field for a specific p-value. Using the variational principle, we determine the equilibrium equation for the magnetization, and solve it for both transitions and at the tricritical point, in order to determine the magnetization profile with respect to h_0.

1. Introduction

A pure model system can describe an experimental sample in rare situations, since such a sample may contain impurities, broken bonds, defects, etc. [1,2], thus necessitating the modification of the pure model appropriately for comparing reliably the theoretical predictions with the experimental results. A small amount of quenched randomness can influence significantly the phase transitions, replacing a first-order phase transition (FOPT) by a second-order phase transition (SOPT), with the result that the tricritical points and the critical end-points are suppressed [3]. In two dimensions, an infinitesimal amount of field randomness destroys any FOPT [3]. The simplest system exhibiting a tricritical phase diagram in the absence of randomness is the Blume–Capel model [4–6]. The random system considered is the one in which random magnetic fields are present, acting on each spin of the system, whose pure version is described by the Ising model; the system in the presence of such fields is called the random field Ising model (RFIM) [7–9]; it has been studied for many years since the seminal work by Imry and Ma [9]. Associated with this model are the notions of lower critical dimension (the lowest spatial dimension above which long range order is established), tricritical points, scaling laws, crossover phenomena and the random field probability distribution function (PDF). The RFIM Hamiltonian is

$$H = -J \sum_{\langle i,j \rangle} S_i S_j - \sum_i h_i S_i, \quad S_i = \pm 1. \tag{1}$$

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The summation in the first term extends over all nearest neighbors and is denoted by $\langle i, j\rangle$; in the second term, h_i represents a random field that couples to the one-dimensional spin variable S_i. The Hamiltonian describes the competition between the long range order (expressed by the first summation) and the random ordering fields. We also consider that $J > 0$, so the ground state is ferromagnetic in the absence of random fields. The presence of random fields requires two averaging procedures, obtaining the usual thermal average, denoted by angular brackets $\langle \ldots \rangle$, and the disorder average over the random fields denoted by $\langle \ldots \rangle_h$ for the respective PDF, which is usually a version of the bimodal distribution or Gaussian distribution.

The most frequently used PDF's for random fields are the bimodal and single Gaussians; the former is

$$P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + h_0) \tag{2}$$

where p is the fraction of lattice sites having a magnetic field h_0, while the rest of the sites have a field $(-h_0)$ with site probability q such that $p + q = 1$ and the usual choice was $p = q = \frac{1}{2}$, the symmetric case, in conjunction with the mean field approximation [10–12]. The latter PDF is

$$P(h_i) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left[-\frac{h_i^2}{2\sigma^2}\right] \tag{3}$$

with zero mean and standard deviation σ [13].

Recently, the asymmetric bimodal PDF (2), $p \neq q$, in general, has also been studied in detail [14]. This study has revealed that for small or large values of p, $p < \frac{13-\sqrt{13}}{26}$ or $p > \frac{13+\sqrt{13}}{26}$, respectively, the paramagnetic/ferromagnetic (PM/FM) boundary is exclusively of second order; in contrast, for $\frac{13-\sqrt{13}}{26} \leq p \leq \frac{13+\sqrt{13}}{26}$ this boundary consists of two branches, a second-order one, for small h_0's and high T's, and another of first order for higher/smaller h_0's and smaller T's; these are joined by a tricritical point (TCP), consequently confirming the existence of such a point under certain circumstances. In addition to these findings, re-entrance phenomena have been observed for specific values of p and h_0 as well as complex magnetization profiles with respect the random field strength h_0. For $p = q = 1/2$, the symmetric bimodal PDF, the results found by Aharony within the mean field approximation were confirmed [10], as well.

Occasionally, the aforementioned distributions are modified to meet the requirements for other circumstances; the bimodal is replaced by the trimodal,

$$P(h_i) = p\delta(h_i - h_0) + q\delta(h_i + h_0) + r\delta(h_i), \tag{4}$$

with $p + q + r = 1$ [15]; the new site probability r is the fraction of sites in the sample not exposed to the random field and usually assumed as $p = q = (1 - r)/2$ in conjunction with mean field approximation [16,17]. The third peak, introduced in addition to the other two in the bimodal (2) and associated with the third term in (4), is to allow for the presence of non-magnetic spins or vacancies in the lattice and results in reducing the randomness of the system.

Another choice for a PDF is the double Gaussian instead of the single Gaussian [18],

$$P(h_i) = \frac{1}{2}\frac{1}{(2\pi\sigma^2)^{1/2}}\left\{\exp\left[-\frac{(h_i - h_0)^2}{(2\sigma^2)}\right] + \exp\left[-\frac{(h_i + h_0)^2}{(2\sigma^2)}\right]\right\}. \tag{5}$$

The two parts of the PDF are equally probable, since $p = q = 1/2$.

For the bimodal/trimodal PDF's, we also make additional assumptions concerning the random field moments:

$$\langle h_i\rangle_h = (p - q)h_0, \qquad \langle h_i h_j\rangle_h = h_0^2\delta_{ij}. \tag{6}$$

The former relation in (6) takes into consideration the probable different site probabilities p and q, whereas the latter implies that there is no correlation between h_i at different lattice sites. Also, the former relation implies the existence of residual magnetism within the system influencing the system's magnetization [14,15].

One of the main issues was the experimental realization of random fields. Fishman and Aharony [19] showed that the randomly quenched exchange interaction Ising antiferromagnet in a uniform field H is equivalent to a ferromagnet in a random field with the strength of the random field linearly proportional to the induced magnetization. This identification gave new impetus to the study of the RFIM; the investigation gained further interest and was intensified, resulting in a large number of publications (theoretical, numerical, Monte Carlo simulations and experimental) in the last thirty years. Although much effort had been invested in this direction, the only well-established conclusion drawn was the existence of a phase transition for $d \geq 3$ (d: space dimension), that is, the critical lower dimension d_l is 2, decided after a long controversial discussion [9,20], while many other questions are still unanswered; among them are those of the order of the phase transition (first or second order), the universality class and the dependence of these points on the form of the random field PDF. Galam, via the MFA, has shown that the Ising antiferromagnets in a uniform field with either a general random site exchange or site dilution have the same multicritical space as the random field Ising model with the bimodal PDF [21]. The study of RFIM has also highlighted another feature of the model, that of tricriticality and its dependence on the assumed distribution function of the random fields. According to the mean field approximation (MFA), the choice of the random field distribution can lead to a continuous FM/PM boundary as in the single-Gaussian probability distribution, whereas for the bimodal one this boundary is divided into two parts, an SOPT branch for high temperatures and an FOPT branch for low