Physica A 390 (2011) 3320–3328

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Theoretical study of the decaying convective turbulence in a shear-buoyancy PBL[☆]

C.J. Kipper[a,*], A. Goulart[b], G. Degrazia[c], M.T. Vilhena[d], P.M. Soares[e]

[a] Departamento de Engenharia Mecânica, Universidade Federal de Santa Maria, 97105-900 Santa Maria, RS, Brazil
[b] Centro de Tecnologia de Alegrete, Universidade Federal do Pampa, 97546-550 Alegrete, RS, Brazil
[c] Departamento de Física, Universidade Federal de Santa Maria, 97111-900 Santa Maria, RS, Brazil
[d] Instituto de Matemática, Universidade Federal do Rio Grande do Sul, 97111-900 Porto Alegre, RS, Brazil
[e] Centro de Geofísica, Universidade de Lisboa, 17049-016 Lisboa, Portugal

ARTICLE INFO

Article history:
Received 13 April 2011
Available online 10 May 2011

Keywords:
Energy spectrum
Planetary boundary layer
Heisenberg's model
Adomian method

ABSTRACT

The derivation of a theoretical model for the decaying convective turbulence in a shear-buoyancy planetary boundary layer is considered. The model is based on the dynamical equation for the energy density spectrum in which the buoyancy, mechanical and inertial transfer terms are retained. The parameterization for the buoyancy and mechanical terms is provided by the flux Richardson number. Regarding the inertial term an approach employing Heisenberg's spectral transfer theory is used to describe the turbulence friction, caused by small eddies, responsible for the energy dissipation of the large eddies. Therefore, a novelty in this study is to utilize the Adomian decomposition method to solve directly without linearization the energy density spectrum equation, with this the nonlinear nature of the problem is preserved. Therefore, the errors found are only due to the parameterization used. Comparison of the theoretical model is performed against large-eddy simulation data for a decaying convective turbulence in a shear-buoyancy planetary boundary layer. The results show that the existence of a mechanical turbulent driving mechanism reduces in an accentuated way the energy density spectrum and turbulent kinetic energy decay generated by the decaying convective production in a shear-buoyancy planetary boundary layer.

1. Introduction

The turbulent transport in the planetary boundary layer (PBL) is a complex physical phenomenon, which is in a process of continuous development driven by the effect of distinct forcing mechanisms [1]. A particular situation characterized by this evolving turbulence is associated with the transition process that happens daily in the layer at sunset. In fact, at the end of the afternoon, the surface heat flux progressively decreases and, then becomes negative and, consequently, a stable boundary layer (SBL) develops near the ground [2,29], while the remnant part of the former convective boundary layer (CBL) starts to decay above this layer. The decay of energy-containing eddies in the CBL is the physical mechanism that can maintain the dispersion process efficient. Turbulence decay in the CBL has been studied by Nieuwstadt and Brost [3] and Sorbjan [4] employing LES models and by Goulart et al. [2] using the energy density spectrum (EDS) dynamical

☆ This work was partially supported by CAPES and Cnpq.
* Corresponding author. Tel.: +55 55 91267272.
E-mail addresses: carlakipper@mail.ufsm.br, carlakipper@gmail.com (C.J. Kipper).
URL: http://www.ufsm.br/ (C.J. Kipper).

0378-4371/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2011.04.019

equation. Furthermore, Caughey and Kaimal [5], Grant [6], Acevedo and Fitzjarrald [7], Grimsdell and Angevine [8] and Anfossi et al. [25] discussed some observational results about this sunset transition time. Therefore, a manner of studying the turbulence decay phenomenon in the CBL is based on the turbulent kinetic energy (TKE) dynamical equation. In this method the unknown terms describing the main turbulent processes must be parameterized. At present, no attempt has been accomplished to include all these terms, namely, the decay of energy-containing eddies in the CBL, the mechanism of inertial transfer of energy from large to small eddies, the shear-driven turbulence generation, and turbulence destruction or generation by buoyancy. Goulart et al. [2] developed a theoretical model to study the decaying of non-isotropic turbulence in a CBL. This model is also based on the EDS dynamical equation in which the buoyancy and energy inertial transfer terms are retained. In the present study, differently from Goulart's paper, we consider shear-driven turbulence generation, buoyancy and energy inertial transfer terms to investigate the convective turbulence decay process of a shear-buoyancy PBL. Furthermore, the method employs Heisenberg's spectral transfer theory to parameterize the mechanism of inertial transfer of energy. This approach allows obtaining an integro-differential equation which is solved using the Adomian decomposition method [9]. This solution method has been shown to solve effectively and accurately a large class of linear and nonlinear, ordinary or partial, deterministic or stochastic differential equations [10,27,11]. Such method provides solutions that converge rapidly to accurate results. The method was good to solve turbulence problems since it does not require unnecessary linearization, perturbation, and other restrictive methods and assumptions which may change drastically sometimes the problem being solved.

2. Turbulent kinetic energy equation in the spectral form

It is possible to derive an equation for the energy spectrum function in a turbulent flow from the momentum conservation law, expressed through the Navier–Stokes equations. In the case of homogeneous turbulent flow, the TKE Fourier transform of the dynamical equation for EDS reads:

$$\frac{\partial E_{i,i}(k,t)}{\partial t} = -M_{i,i}(k,t)\frac{\mathrm{d}\overline{U}_i}{\mathrm{d}x_j} + \frac{g}{\theta}H_{i,\theta}(k,t) + W_{i,i}(k,t) - 2\nu k^2 E_{i,i}(k,t), \tag{1}$$

where i represents the longitudinal (u), lateral (v) and vertical (w) velocity components, t is the time, k is the wavenumber, $\frac{g}{\theta}$ is the buoyancy parameter with g representing the local gravity acceleration and θ is the temperature, $E_{i,i}(k,t)$ is the three-dimensional (3D) EDS, $M_{i,i}(k,t)\frac{\mathrm{d}\overline{U}_i}{\mathrm{d}x_j}$ is the energy production by mechanical (shear) effect, $W_{i,i}(k,t)$ is the transport term that represents the contribution due to the inertial transfer of energy among different wavenumbers, $H_{i,\theta}(k,t)$ is a term of production or loss due to buoyancy contribution and the last term represents the energy dissipation by molecular viscosity.

2.1. Parameterization of the mechanical and convective terms

In order to parameterize the mechanical and convective terms in the Eq. (1), an useful approximation can be made employing the flux Richardson number. It is defined as the ratio between the term of production or loss of energy by thermal effect and the term of energy production by mechanical effect. The ratio of the source terms of convective and mechanical turbulence is therefore given by the flux Richardson number

$$R_f = \frac{\frac{g}{\theta}H_{i,\theta}(k,t)}{-M_{i,i}(k,t)\frac{\partial\overline{U}_i}{\partial x_j}}. \tag{2}$$

In the convective regime the buoyancy term creates a transfer of energy upwards (positive) and the mechanical production term generates an energy transfer downwards (negative). Through the expression (2), one can establish a relationship between the terms of buoyancy and mechanical effect represented in the equation below

$$\frac{g}{\theta}H_{i,\theta}(k,t) - M_{i,i}(k,t)\frac{\partial\overline{U}_i}{\partial x_j} = -M_{i,i}(k,t)\frac{\partial\overline{U}_i}{\partial x_j}(R_f+1). \tag{3}$$

Considering a homogeneous flow invariant against the translation property, it is possible to express the rate of the characteristic strain $\frac{\partial\overline{U}_i}{\partial x_j}$ of each vortex [12] within the wavenumber, from a Fourier transform, as follows

$$\frac{\partial\overline{U}_i}{\partial x_j} = \frac{[k^3E_{i,i}(k,0)]^{\frac{1}{2}}}{2\pi}. \tag{4}$$

The interaction between eddies of different sizes and the rate of deformation of the mean flow provides a flow of energy that causes the production of mechanical energy. Such process is described by [12]

$$P(k,t) = \left(\frac{\partial\overline{U}_i}{\partial x_j}\right)E_{i,i}(k,0)\cos(\beta t), \tag{5}$$