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A unified model for price return distributions used in econophysics

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ABSTRACT

For a decade, a new theoretical movement called "econophysics" has been initiated by some physicists who began to publish articles devoted to the study of economic and financial phenomena. Since then, econophysicists have written a very prolific literature about the way of characterizing the evolution of financial prices. Today, there is an "extreme diversity" of models recently developed by econophysicists whose research is sometimes presented as an ill-defined field. The objective of this paper is precisely to provide a unified framework in order to contribute to unify econophysics and to base this new field on shared scientific standards.

0. Introduction

For the 1970s, a new theoretical movement has been initiated by some physicists who began publishing articles devoted to the study of social phenomena, such as the formation of social groups [1] or social mimetism [2].[1] The next decade confirmed this new theoretical trend (labelled *sociophysics*[2]), as the number of physicists publishing papers devoted to the explanation of social phenomena and the number of themes analysed continued to increase.[3] During the 1990s, physicists[4] turned their attention to economics, and particularly financial economics, giving rise to econophysics.[5] Although the movement's official birth for example announcement came in a 1996 article by Stanley et al. [11],[6] econophysics was at that time still a young and ill-defined field. Econophysics can be defined as "a quantitative approach using ideas, models,

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1 Regarding the emergence and history of sociophysics, see Ref. [3].

2 This term was proposed by Serge Galam in a 1982 article. In his view, one of the reasons why physicists attempt to explain social phenomena stems from a kind of mismatch between the theoretical power of physics and the inert nature of its subject matter: "During my research, I started to advocate the use of modern theory phase transitions to describe social, psychological, political and economical phenomena. My claim was motivated by an analysis of some epistemological contradictions within physics. On the one hand, the power of concepts and tools of statistical physics were enormous, and on the other hand, I was expecting that physics would soon reach the limits of investigating inert matter" [2, p. 50].

3 Let us mention, for example industrial strikes [4], democratic structures [5], and elections [3,6].

4 The influence of physics on the study of financial markets is not new, as witnessed by the work of Bachelier [7] and Black and Scholes [8]. Nevertheless, we cannot yet refer to Black & Scholes' model as econophysics in the term's current meaning, since it was completely integrated into the dominant theoretical current of economics and finance [9]. Econophysics is not an "adapted import" of the methodology used in physics; rather, it is closer to a "methodological invasion". We return to this point in the next section.

5 For a historical analysis, see Ref. [10].

6 This article is also the origin of the term econophysics.
We would point out, however, that Kutner and Grech [12] trace the informal birth of the approach to the paper by Mategna [13] that studied the evolution of returns on financial markets in terms of Lévy processes. This definition seemed to gain ground as a compromise, and is found in a number of books and articles produced by the current, for example by Wang et al. [14, p. 1] or Rickles [15].

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conceptual and computational methods of statistical physics''.[7] Today, econophysics is an institutionalized field, [17] with different journals proposing a prolific literature about the way of characterizing the evolution of financial prices. There is an ''extreme diversity'' of models recently developed by econophysicists [18] and many theoretical frameworks still emerge.

In this paper, our objective is precisely to provide a such unified framework. Indeed, the standardization of knowledge through a common scientific culture is a necessary condition to become a strong discipline [19]. We propose a generic formula characterizing the statistical distributions usually used by econophysicists (Levy, Truncated Levy or no stable Levy distributions). Such formula will contribute to unify econophysics and to base this new field on shared scientific standards since the possibility to find a generalized formula is derived from the common conceptual tools shared by econophysicists. This will enable econophysics be no longer an ill-defined field. Moreover, such generalized formula allows a systematic comparison between the different models used by econophysicists.

1. Econophysics: a new field of research

According to Kutner and Grech [12], econophysics as a field of research dates back to 1991 when Mantegna published a paper about Levy process in finance. However, one can trace the roots of the basic ideas of econophysics to papers by Benoît Mandelbrot [20,21] who saw an analogy between the evolution of financial markets and the phenomenon of turbulence. It is only about thirty years later that these discussions re-emerged under the label ''econophysics''. As the name suggests, econophysics presents itself as a hybrid discipline which can be defined in methodological terms as ''a quantitative approach using ideas, models, conceptual and computational methods of statistical physics'' applied to economic and financial phenomena [22, p. 1].

Econophysics presents itself as a new way of thinking about the economic and financial systems through the ''lenses'' of physics [23]. As much as neoclassical economics imported models from classical physics as formulated by Lagrange [24], and financial economics built on the model of Brownian motion also imported from physics, econophysics tries to model economic phenomena using analogies taken from modern condensed matter physics and its associated mathematical tools and concepts. Using the standard tools of statistical mechanics including microscopic models like Ising model and scaling laws, econophysicists aim at explaining how complex economic systems behave. Broadly speaking, econophysics is founded on general statistical properties that reappear across many and diverse phenomena [25]. This statistical regularity can be characterized by scaling laws that are considered as the heart of econophysics[8] [26] or [27, p. 288]. These scaling laws can take a variety of forms. The objective of the next section is to offer a generic formula characterizing the main distributions usually used by econophysicists.

2. Generalized formula for price return distributions

For describing the probability distributions of stock market price changes, many models using different types of probability functions are proposed in the econophysics literature. However, Gringras and Schinckus [17] showed that *Physica A* appears to be the leading journal and that Mantegna, Bouchaud, Mandelbrot, Sornette and Lux are the most cited authors in econophysics. Our analysis is based on these results. We also add other important authors such as Stanley, Gopikrishnan or Plerou who are also very cited authors in econophysics (*Web of Science*). Among the authors identified, we have selected econophysics papers dedicated to distribution of price returns (see Tables 1 and 2). From articles listed in Tables 1 and 2, we propose the following generalized formula:

$$P(x) = Cf(x)e^{-g[h(x)]+d} \tag{1}$$

where, C and d are constants that might have temporal variation.

The analytical form of $f(x)$ is not always known for all the values of x, but it has a power law variation in the limit of large $x(x \to \infty)$:

$$f(x) \sim \frac{1}{x^{b_1+a_1\alpha}} \quad \text{for } |x| \to \infty \tag{2}$$

a_1 and b_1 are two parameters (usually equal to 1) that define the shape of the distribution at large x, and α is the principal exponent of the power law. The function g introduced in Eq. (1) has the form:

$$g(x) = (a_2h(x) + b_2)^{c_2} \tag{3}$$

7 [11, p. 2] To present econophysics as an extension of statistical mechanics necessitates a better definition of this approach in physics. Statistical mechanics attempts mainly to explain in statistical terms the behaviour and macroscopic evolution of a complex system on the basis of interactions of a large number of microscopic constituents (atoms, electrons, ions, etc.) that make it up [16, p. 155]. Applied to finance, this type of reasoning allows one to consider the market as the statistical and macroscopic results of a very large number of heterogeneous interactions at the microscopic level.

8 These scaling laws can then be viewed as a macroresult of the behaviour of a large number of interacting components from lower levels. As Rickles [18] explains, ''The idea is that in statistical physics, systems that consist of a large number of interacting parts often are found to obey 'universal laws' – laws independent causally of microscopic details and dependent on just a few macroscopic parameters''.