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Nonextensivity measure for earthquake networks

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HIGHLIGHTS

- We show the q-exponential appears as an appropriate function for fitting the degree distribution of the earthquake networks.
- We show that the q-exponent as a function of resolution has a peak.
- This peak represents a threshold for our previous assertion on dependence of the network's characteristics on the resolution.

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ABSTRACT

Studying earthquakes and the associated geodynamic processes based on the complex network theory enables us to learn about the universal features of the earthquake phenomenon. In addition, we can determine new indices for identification of regions geophysically. It was found that earthquake networks are scale free and its degree distribution obeys the power law. Here we claim that the q-exponential function is better than power law model for fitting the degree distribution. We also study the behavior of q parameter (nonextensivity measure) with respect to resolution. It was previously asserted in Eur. Phys. J. B (2012) 85: 23; that the topological characteristics of earthquake networks are dependent on each other for large values of the resolution. A peak in the plot of q against resolution determines the beginning of the assertion range.

1. Introduction

Nowadays complex network theory is widely used to study complex phenomena in many disciplines. Earthquakes display a complex spatio-temporal behavior of the earth's crust. We can quantify the complexity of the earthquake phenomena by computing its associated network characteristics. In the first step we should describe how to construct earthquake networks.

Baiesi and Paczuski introduced a metric which takes into account the time interval, special distance between two earthquakes and almost the magnitude of the first event for quantifying the correlation between two seismic events. In this definition, linked nodes are strongly correlated pairs, and an earthquake network is the set of all linked nodes [1]. They found that such a network is scale free with exponent $\gamma \approx 2$.

Telesca and Lovallo investigated properties of a network of Italian earthquakes by means of the visibility graph method [2]. They found that the degree distribution of the magnitude point process is power law. It is also discovered that with the increase of the magnitude threshold, degree distribution does not change significantly.

Abe and Suzuki used an alternative way for constructing earthquake networks, by dividing the specific geographical region into small cubic or square cells. If an earthquake with any values of magnitude occurs in a cell, they identify it as a vertex of a network. Two successive events in vertices are connected to each other by an edge. Then, different properties of such networks in California and Japan have been studied [3–8]. Furthermore, it has been shown that it is of the small-world [9] and scale free type [10]. One of the astonishing results is concerned with the clustering coefficient that remains

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stationary before main shocks. At the main shocks, it has a sudden jump, and then it decays slowly to become stationary again [7,8]. The clustering structure of an earthquake network has been studied recently [11]. Further, the dependence of this value on the size of real seismic data has been examined. It has been discovered that there exists a scaling law for the clustering coefficient in terms of cell-size which was needed for constructing a network. It has been found that scaling function associated with the clustering coefficient approaches to an invariant value when the cell-size becomes larger than a certain value. Dependence of the scale free exponent, γ; to the cell-size was almost discovered [12]. It has been reported that the γ approaches a fixed value and the cell-size dependence disappears when the cell-size becomes larger than a certain value. In addition, the dependence of the clustering coefficient to the cell-size has been considered too. Again, it also takes the universal invariant value ≈ 0.85 as the cell-size becomes larger than a certain value.

In our recent work [13], we have studied the role of the resolution; a parameter that is related to the inverse of the cell size, on the features of earthquake networks in Iran and California. For large values of the resolution, we showed that all network characteristics behave as a power law function of resolution and by the reason are dependent on each other. Thus one of the topological characteristics is adequate for describing the earthquake network.

In this work we show the q-exponential appears as an appropriate function for fitting the degree distribution of earthquake networks. We also show that the q-exponent as a function of resolution has a peak. This peak represents a threshold for validity of our previous assertion.

This paper is organized as follows, Section 2 is devoted to describe the nonextensive statistical mechanics in brief, in Section 3 we review the method of constructing earthquake networks and the results are presented in Section 4.

2. Nonextensive mechanics

Tsallis in 1988 generalized the traditional definition of the Boltzmann–Gibbs for the entropy [14],

$$S_q = \frac{\left(1 - \sum_{i=1}^{\Omega} p_i^q\right)}{q-1}. \tag{1}$$

in which p_i represents the probability for occurrence of the i-th micro-state of the system and Ω shows the total accessible number of micro-states. q is a parameter that exhibits the internal properties of system. Subsequently it was discovered that the Tsallis entropy can describe a wide range of the complex phenomena [15].

An earthquake is a complex and nonlinear phenomenon in the earth's crust. The nonlinearity may result in partitioning the phase space so that the state of the system is restricted to be in particular region of the phase space at least for a finite period of time. In such a case we cannot extract complete information about systems micro-states by considering the time average of the event sequence. It is believed the nonextensive statistical mechanics is a good candidate for dealing with systems that exhibit incompleteness. The Tsallis entropy puts this incompleteness in the parameter q. In the context of nonextensive statistical mechanics, the probability of micro-states is related to time averaging analysis by defining the escort probability [16].

Two factors are the reasons for incompleteness in seismic time series, the problem with monitoring small earthquakes and also the finite length of data. Therefore, it is rational to use Tsallis entropy for studying earthquake networks. In this case, $p_i = p(k_i)$ is the probability that a node has a k_i link. p_i is not an observable because some events may possibly remain obscured. We can define the actual probability, π_i, associated to any probability p_i which is called the escort probability.

$$\pi_i = \frac{p_i^q}{\sum_{i=1}^{\Omega} p_i^q}. \tag{2}$$

The escort probability is measured from empirical data and obeys the normalization condition.

$$\sum_{i=1}^{\Omega} \pi_i = 1. \tag{3}$$

The nonextensive statistical mechanics is constructed by maximization of the Tsallis entropy accompanied by the normalization condition and other constraints in the system, as,

$$\sum_{i=1}^{\Omega} k_i \pi_i = K. \tag{4}$$

K is a constant. The result is probability density,

$$p_i \sim (1-(1-q)\beta k_i)^{\frac{q}{(1-q)}}, \tag{5}$$

β is the reduced Lagrange multiplier. For practical purpose, it is better to work with cumulative probability,

$$P_i = \sum_{j=i}^{\Omega} \pi_j. \tag{6}$$

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