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The additive property of the inconsistency degree in intertemporal decision making through the generalization of psychophysical laws

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ABSTRACT

Intertemporal decision making involves choices among options whose effects occur at different moments. These choices are influenced not only by the effect of reward value perception at different moments, but also by the time perception effect. One of the main difficulties that affect standard experiments involving intertemporal choices is the simultaneity of both effects on time discounting. In this paper, we unify the psychophysical laws and discount value functions using the one-parameter exponential and logarithmic functions from nonextensive statistical mechanics. Also, we propose to measure the degree of inconsistency. This quantity allow us to discriminate both effects of time and value perception on discounting process and, by integration, obtain other main quantities like impulsivity and discount functions.

1. Introduction

In contrast to Physics, Economics is based on several axioms and only in the last decades have they been extensively explored by observation. This may lead to limitations and deviations when standard economic models are tested empirically [1]. Most of the revision of these problems and the formulation of new models involve an interdisciplinary context.

Intertemporal decision making involves choices among options whose effects occur at different moments. The implications of these choices on everyday activities led to the search of its underlying principles. Mathematical functions that fairly describe the time discount process have been suggested by experiments. These experiments involve humans and non-human animals and are influenced by several factors of variability. The consensus is that delayed rewards are discounted (or undervalued) relative to immediate rewards [2].

The discount process may first be assigned to changes in perception (evaluation) of a reward value at different moments. However, individuals, when forming their intertemporal preferences, may estimate time intervals in a non-objective manner [3–6]. Thus, the discount process in intertemporal choices comprises not only the effect of reward value perception at different moments, but also the time perception effect.

One of the main difficulties in determining discount functions from experiments is the simultaneity of both effects on time discounting. The independent analysis of each factor is not allowed by standard experiments that directly measure these functions.

From discount functions one can obtain other quantities validated by experiments. The *impulsivity* measures the strong preference for immediate rewards over delayed ones, even though the magnitude of the delayed reward is more

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advantageous. Also, individuals tend to prefer smaller immediate rewards in the near future (reflecting impulsivity) but tend to prefer larger later rewards in the distant future. This preference reversal over time is referred as *inconsistency* in intertemporal choices [7].

In this paper we unify the Weber–Fechner and Stevens psychophysical laws using the one-parameter exponential and logarithmic functions from nonextensive statistical mechanics [8]. This allows us to propose new general discount value functions. The determination of the intricate dependence between value and time perception effects in the discount function may be softened exploring their additiveness in the degree of inconsistency. Since value and time perception are additive in the inconsistency degree, experiments may be designed to measure them independently. By integration of the degree of inconsistency, one obtains the impulsivity and discount functions. The standard experiments used in the context of intertemporal decision making need to be reformulated for better understanding of the governing processes.

This study is outlined as follows. In Section 2 we present an overview of usual experiments and some theoretical models in intertemporal decision making. In Section 3 we describe our main findings and proposals for a new class of experiments based on the inconsistency degree. Finally, the conclusions are presented in Section 4.

2. Intertemporal decision making: theory and experiments

This section presents an overview of concepts and results of the literature involving intertemporal decision making. We start describing the *discount functions* and the standard experiments in intertemporal choices. We present two theoretical models that aim to describe the time discounting process observed in experiments: the exponential and the hyperbolic models. Next, we introduce the *impulsivity* and the *inconsistency*, which provide basic tools to compare exponential and hyperbolic discount models. Other theoretical models are also addressed. Finally, we describe the *psychophysical laws* and the association of the so-called *psychophysical effects of time perception* to the temporal discounting models.

2.1. Discount functions

Intertemporal choices refer to choices between options (rewards) whose consequences occur at different times. Individuals subjected to intertemporal choices face a conflict (*trade off*) between the utility (or value) of an immediate reward and a delayed one. Consider the following examples: choosing between \$10 today or \$15 in a month; choosing to spend all earnings today or to save money for the future; deciding whether smoke or not a cigarette, to preserve health. In intertemporal choices, the time interval between the present time and the time when the reward is delivered is referred as *delay* [9].

Many studies have led to a strong consensus that delayed rewards (V) are discounted (or undervaluated) relative to immediate rewards (V_0) [2]. The value (or utility) of a reward V decreases as the time interval until its receipt (t) increases. The non-discounted (real) value of a given reward is called *objective value*. The value to be received immediately, which is equivalent to the receipt of V_0 on a specified delay, is referred as the *subjective value* of the reward or *indifference point*. The subjective value behavior of a reward as a function of delay, $V(t)$, is analyzed throughout the *discount functions*. The shape of the discount curve is a decreasing monotonic function with null asymptotic value.

2.1.1. Experiments

Experiments, with both humans and nonhuman animals, have been conducted to determine the indifference points [10, 7,11–15]. In general, in the experiments involving delay discounting with humans, the participants choose between two monetary rewards, a smaller but immediate reward and another of greater value delivered after a given delay. For each delay, the experiment begins with equal values for both rewards, so that a given participant chooses the immediate reward. The delayed reward value is kept constant while the immediate reward value is decreased. Next, the participant performs a new decision-making between immediate and delayed rewards. This procedure is repeated until the delayed reward is preferred to the immediate one. The last immediate reward value chosen, V_d, is described as the indifference point of the respective delay. To avoid a possible influence of the rewards presentation order in the experiments, the reverse procedure is also examined. The reversed experiment starts from the lowest value for the immediate reward, so that the delayed reward is preferred. The immediate reward is then increased until its first value, V_s, is chosen. The indifference point is obtained from the average between V_d and V_s. The indifference points obtained for different delays are fitted and described by discount functions.

In most experiments involving intertemporal choices hypothetical rewards are used. Also, the delays are not experienced by the individual during the experiment. This type of procedure has the advantage of being cheap and time efficient.

To check the results validity, few studies have compared experimental data for procedures involving hypothetical and real rewards. In the latter, a response is randomly selected among the choices made by the participant, so that one can receive a real reward, according to value and delay chosen [16]. Up to Johnson and Bickel study [17], no experiment analyzed the same participants in both conditions (real and hypothetical). In their study, no significant differences have been observed between real and hypothetical procedures. However, one must consider that the reward values and delays used in the real experiments were smaller than those used in hypothetical procedures. Madden et al. [18] analyzed the same reward values