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Quantum spherical model with competing interactions

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ABSTRACT

We analyse the phase diagram of a quantum mean spherical model in terms of the temperature T, a quantum parameter g, and the ratio $p = -J_2/J_1$, where $J_1 > 0$ refers to ferromagnetic interactions between first-neighbour sites along the d directions of a hypercubic lattice, and $J_2 < 0$ is associated with competing antiferromagnetic interactions between second neighbours along $m \leq d$ directions. We regain a number of known results for the classical version of this model, including the topology of the critical line in the $g = 0$ space, with a Lifshitz point at $p = 1/4$, for $d > 2$, and closed-form expressions for the decay of the pair correlations in one dimension. In the $T = 0$ phase diagram, there is a critical border, $g_c = g_c(p)$ for $d \geq 2$, with a singularity at the Lifshitz point if $d < (m+4)/2$. We also establish upper and lower critical dimensions, and analyse the quantum critical behavior in the neighborhood of $p = 1/4$.

1. Introduction

The spherical model of magnetism has been used as an excellent laboratory to test ideas and concepts of phase transitions and critical phenomena [1–3]. There are several versions of the original model, including proposals of a quantum spherical model to correct some of the unphysical results at low temperatures [4–7]. The effects of frustration [8,9], random fields [10], and of disordered exchange interactions [11,12], have also been analysed in the context of quantum spherical models. With a view to describe the crossover between classical and quantum critical behaviour, Vojta [6] used a standard scheme of canonical quantization to analyse a quantum version of the ferromagnetic mean spherical model. We were then motivated to revisit this problem, with the addition of competing ferro and antiferromagnetic interactions, and the perspective to analyse a quantum Lifshitz point.

The mean spherical model, which has been originally proposed by Lewis and Wannier [2], is given by the partition function

$$Z_{cl} = \left[\prod_{\vec{l}} \int_{-\infty}^{+\infty} dS_{\vec{l}} \right] \exp\left[-\beta\, \mathcal{H}\left\{S_{\vec{l}}\right\} - \beta\mu \sum_{\vec{l}} S_{\vec{l}}^2 \right], \tag{1}$$

where $\beta = 1/(k_B T)$, T is the temperature and k_B is the Boltzmann constant, μ is a suitable chemical potential, $\vec{l}$ is a lattice vector, and $\left\{S_{\vec{l}}\right\}$ is a set of continuous spin variables running over the N^d sites of a d-dimensional hypercubic lattice. The model Hamiltonian is written as

$$\mathcal{H} = -\sum_{\left(\vec{k},\vec{l}\right)} J_{\vec{k},\vec{l}}\, S_{\vec{k}} S_{\vec{l}} - H \sum_{\vec{l}} S_{\vec{l}}, \tag{2}$$

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where $\left(\vec{k},\vec{l}\right)$ labels a pair of lattice sites, the exchange parameter $J_{\vec{k},\vec{l}} = J\left(\left|\vec{k}-\vec{l}\right|\right)$ depends on the distance between sites $\vec{k}$ and $\vec{l}$, and H is an external field. In this formulation, the chemical potential μ comes from the mean spherical condition,

$$\left\langle \sum_{\vec{l}} S^2_{\vec{l}} \right\rangle = -\frac{1}{\beta}\frac{\partial}{\partial\mu}\ln Z_{cl} = N^d, \tag{3}$$

and it is well known that exact solutions for the thermodynamic functions can be obtained from the standard diagonalization of a quadratic form [3].

In a quantum version of this mean spherical model [4,6], the spin variable $S_{\vec{l}}$ becomes a position operator at lattice site $\vec{l}$, canonically conjugate to a momentum operator $P_{\vec{l}}$, with the commutation relations

$$[S_{\vec{l}}, S_{\vec{k}}] = 0, \qquad [P_{\vec{l}}, P_{\vec{k}}] = 0, \qquad [S_{\vec{l}}, P_{\vec{k}}] = \mathrm{i}\delta_{\vec{l},\vec{k}}, \tag{4}$$

where $\delta_{\vec{l},\vec{k}}$ is a Kronecker delta and we assume that $\hbar = 1$. We then add a term of kinetic energy, depending on a quantum parameter g, and write the quantum quadratic form

$$\overline{\mathcal{H}} = \frac{1}{2}g\sum_{\vec{l}} P^2_{\vec{l}} - \sum_{\left(\vec{k},\vec{l}\right)} J_{\vec{k},\vec{l}}\, S_{\vec{k}} S_{\vec{l}} - H\sum_{\vec{l}} S_{\vec{l}} + \mu\sum_{\vec{l}} S^2_{\vec{l}}, \tag{5}$$

which can be diagonalised by a canonical method [6], leading to a solution of the problem for a general ferromagnetic pair interaction. At finite temperatures, the critical behaviour is essentially unchanged with respect to the classical spherical model. At zero temperature, depending on the parameter g, there is a quantum phase transition characterised by new (quantum) critical exponents. Also, the introduction of quantum fluctuations leads to a correction of the unphysical behaviour of the entropy at low temperatures.

We report an analysis of this version of the quantum mean spherical model in the presence of competing interactions. We consider ferromagnetic interactions, $J_1 > 0$, between pairs of first-neighbour sites along the d directions of a hypercubic lattice, and antiferromagnetic interactions, $J_2 < 0$, between second-neighbour sites along $m \leq d$ directions. Classical versions of this model [13–16], as well as more elaborate mean spherical models with competing interactions [17], have been studied by several authors. For $m = 1$, we regain a spherical analogue of the Axial-Next-Nearest-neighbour Ising, or ANNNI, model [18,19], which is known to display a rich phase diagram, including a Lifshitz point, in terms of the temperature T and a parameter $p = -J_2/J_1$ that gauges the strength of the competing interactions. We then analyse the T–p–g phase diagram, for different values of m, in particular the $T = 0$ behaviour, and establish the critical dimensions and critical exponents associated with this quantum model system. In the classical case, $g = 0$, we confirm a number of results, including a singularity of the critical border at the Lifshitz point for $2 < d < (m+6)/2$. In one dimension, we derive analytic expressions for the decay of pair correlations, and determine the region of modulated behaviour in the T–p phase diagram.

2. The quantum mean spherical model with competing interactions

This problem can be treated either by a conventional reduction to a system of coupled harmonic oscillators or by a judicious application of the method of path integrals [5,20,21]. Let us first use the representation in terms of harmonic oscillators. We then introduce bosonic operators $a_l^\dagger$ and a_l to write

$$S_l \equiv \frac{1}{\sqrt{2}}\left(\frac{g}{2\mu}\right)^{1/4}\left(a_l + a_l^\dagger\right) \tag{6}$$

and

$$P_l \equiv -\frac{\mathrm{i}}{\sqrt{2}}\left(\frac{2\mu}{g}\right)^{1/4}\left(a_l - a_l^\dagger\right), \tag{7}$$

where we have omitted the vector notation. We now assume periodic boundary conditions, and change to a Fourier representation,

$$a_l = \frac{1}{N^{d/2}}\sum_q a_q \exp(\mathrm{i}ql), \tag{8}$$

where a_q and $a_q^\dagger$ are bosonic operators,

$$[a_q, a_{q'}] = 0, \qquad [a_q^\dagger, a_{q'}^\dagger] = 0, \qquad [a_q, a_{q'}^\dagger] = \delta_{q,q'}, \tag{9}$$