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Numerical studies of an interacting particle system and its deterministic description

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Abstract

We present numerical results on a stochastic population dynamics model with reproduction rates depending on the density of particles in a neighborhood. In order to investigate the role of fluctuations some characteristics of the model are compared with those computed from its deterministic (noise-less) density equation. In particular, we study the stationary density of particles when a pattern is developed and the typical size of the clusters of particles.

Keywords: Interacting particle systems; Cluster formation; Pattern formation

1. Introduction

Interacting particle systems mimicking biological individuals competing for resources may provide insights into the dynamics of real populations. To this end, the authors recently introduced [1] a simple model of particles representing *bugs* that reproduce and die at rates depending on the number of particles present within a given distance. The main feature of the model is that, for a specific range of the parameters, the particles arrange in a periodic pattern of clusters. In previous works [1,2] it was shown that this can be explained as an instability in the deterministic

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equation derived for the density of particles. However, some of the properties of the original particle system cannot be explained within the framework of a deterministic density equation because of the fluctuations arising from the discrete character of the particles. Here we briefly present further properties of the model and of the deterministic description that properly establishes the role of fluctuations. Considering the rather general mechanism introduced in the particle interaction (competition with neighbors), the implications of these studies can be extended to more realistic biological population dynamics.

The model consists of $N(t)$ particles at time t performing Brownian motion, with diffusion coefficient D, in the line (in this paper we restrict ourselves to the one-dimensional periodic case: a ring of length L). Each particle i $(=1,\ldots,N)$ reproduces with rate (probability per unit of time) λ_i, giving rise to a newborn which is located at the same place as the parent particle, or dies at rate β_i, disappearing from the system. The death rate $\beta_i \equiv \beta_0$ is a constant and the same for all the particles, but the birth rate λ_i models competition for resources, so that it decreases with the number of neighbors $N^i_R(t)$ that particle i has within a given distance R: $\lambda_i = \lambda_0 - N^i_R(t)/N_s$. λ_0 and N_s are constants. In Ref. [1] the following equation for the expected density of particles $\phi(x,t)$ was derived under the approximation of negligible fluctuations:

$$\partial_t \phi(x,t) = \mu\phi(x,t) + D\nabla^2\phi(x,t) - \frac{1}{N_s}\,\phi(x,t)\int_{|x-y|\leqslant R} \phi(y,t)\,\mathrm{d}y\,, \tag{1}$$

where $\mu = \lambda_0 - \beta_0$. This deterministic integrodifferential equation and similar ones have been directly introduced in related contexts [3,4]. For $\mu > \mu_P = 84.2D/R^2$ its solution tends to a steady spatially periodic pattern at long times, which explains the clustering occurring in the particle system.

In this paper we summarize some numerical and analytical calculations of the stationary density of particles (Section 2), and the size of clusters (Section 3). In both cases, the emphasis is in the comparison of the behavior of the stochastic particle model with the deterministic density approximation (1).

2. Stationary density of particles

Fig. 1 shows (circles) the mean density of particles (defined as the temporal average ρ in the quasisteady state of $\rho(t) = N(t)/L$, the quotient between the total number of particles $N(t)$ and system size L). There is a sharp transition between the absorbing state $\rho = 0$ and the active phase $\rho > 0$ on increasing the parameter $\mu = \lambda_0 - \beta_0$ above a critical value μ_c (≈ 0.34 in the figure). At sufficiently large values of μ the density approaches a nearly linear dependence on μ. The transition behavior given by the numerical solution of the deterministic model (1) is completely different (squares), as stressed in [2]. In particular, the deterministic prediction indicates that the active phase appears above $\mu_c = 0$, which is much below the particle model μ_c. In fact, μ_c is above the deterministically predicted value for μ_P ($\mu_P \approx 0.084$ for the parameters of the figure), consistent with the observation that the active phase in the