Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 355 (2005) 165–175
PHYSICA A
www.elsevier.com/locate/physa

Frequency domain principal components estimation of fractionally cointegrated processes: Some new results and an application to stock market volatility

Claudio Morana[a,b,*]

[a] *University of Piemonte Orientale, Novara, Italy*
[b] *International Centre for Economic Research (ICER), Italy*

Received 31 October 2004; received in revised form 3 January 2005
Available online 4 May 2005

Abstract

In this paper we present new results for the frequency domain principal components estimator of the cointegration space for stationary long memory processes of Morana [Appl. Econ. Lett. 11 (2004) 837], concerning asymptotic properties, identification of the cointegration space and the linkage with the frequency domain least-squares estimator. An application of the approach to stock market volatility data shows that the methodology can effectively be employed for the modelling of long-run relationships, which could not be handled using the standard $I(1)$–$I(0)$ cointegration approach.

PACS: C22

Keywords: Fractional cointegration; Long memory; Frequency domain analysis; Stock market volatility

*Corresponding address: Faculty of Economics, University of Piemonte Orientale, Via Perrone 18, 28100 Novara, Italy.
E-mail address: morana@eco.unipmn.it.

0378-4371/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2005.02.079

1. Introduction

Recently several contributions to the estimation of fractionally cointegrated processes have been made in the literature. Contributions have been concerned with both the estimation of single and multiple cointegrating vectors for mean reverting and non-mean reverting fractionally integrated processes and testing for cointegration. As far as the stationary long memory case is concerned, the *narrow band frequency domain least-squares estimator* (FDLS) has initially been proposed by Robinson [1] for the estimation of single cointegrating vectors, with cointegrating residuals possibly also showing long memory. The estimator has been shown to have superior asymptotic properties relative to the *ordinary least-squares estimator* (OLS), being consistent also in the presence of correlation between the cointegrating residuals and the regressors. Asymptotic normality of the estimator has been proved by Christensen and Nielsen [2] for the case of a collective memory of the regressor and the cointegrating residuals below the long memory stationary boundary. As an alternative to the FDLS estimator, Hualde and Robinson [3] have proposed a *root-n consistent estimator* (RNC) with Gaussian limiting distribution in the case of weak cointegration, which amounts to OLS estimation of the cointegrating vector using a dynamic relation for the variables of interest, close to the error correction representation. A different approach has been proposed by Nielsen [4], consisting of *two-step local Whittle* joint estimation (2SLW) of the order of fractional differencing (of the actual series and the cointegrating residuals) and of the cointegrating vector. The proposed estimator is consistent and asymptotically normal, and relatively asymptotically more efficient than the FDLS estimator, although showing the same rate of convergence of this latter estimator. On the other hand, Morana [5] has proposed system estimation for the case of multiple cointegrating vectors, based on a *frequency domain principal components* approach (FDPC), implemented through the eigenvalue–eigenvector decomposition of the long-run variance–covariance matrix of the fractionally differenced processes. An approach to identify and estimate the common long memory factor structure and to implement a persistent–non-persistent decomposition has also been proposed by Morana [5], and existing theoretical results have been employed to demonstrate the consistency of the estimator of the common long memory factor structure. Monte Carlo simulations comparing the small sample performance of the FDPC, FDLS, 2SLW and RNC estimators for the stationary long memory case are available in Refs. [5] (FDPC and FDLS), [4] (2SLW), and [3] (RNC and OLS). The results suggest that all the estimators are unbiased when the regressor and the cointegrating residuals are not contemporaneously correlated, while the FDPC, FDLS, RNC, and OLS estimators are biased when this property does not hold. In particular, the bias has been found to increase with the degree of correlation between the regressor and the cointegrating residual, and to fall as the strength of cointegration increases, i.e., as the collective memory decreases. Yet, fully unbiased estimation can be achieved by the FDPC and FDLS estimators by applying bias correction factors computed as suggested in Ref. [5]. On the other hand, the performance of the 2SLW estimator when the regressors and the cointegrating residuals are correlated has not yet been