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Unstable growth and anomalous scaling

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HIGHLIGHTS

- A growth model with non-uniform deposition probability is studied.
- Surface morphology presents features that depend on the attachment non-uniformity.
- The model is unstable with characteristics found in anomalous scaling.

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ABSTRACT

We have studied the dynamic scaling properties of a growth model in which the particle attachment probability is a function of surface curvature. Preferential incorporation of particles at protuberances (rather than over valleys) is followed by surface diffusion. Through extensive numerical simulation, the model is found to display unstable growth, whose features depend on the attachment non-uniformity. Strikingly, all the characteristics found in anomalous scaling are also observed in this unstable growth.

1. Introduction

Surface growth processes are widely studied because of their theoretical interest and practical applications in many different fields [1,2]. Analytical and numerical methods developed to understand the physical mechanisms that determine the evolution of the surface morphology during growth, continue being the subject of great attention [3,4].

In many systems, the interface roughness W (root-mean-square deviation around the mean interface position) evolves at short times as a power law of time – $W(t) \sim t^{\beta}$, where β is called the growth exponent – until it eventually reaches a steady-state that scales with the system size L as $W \sim L^{\alpha}$ (where α is the so-called roughness exponent). However in real experiments, for which varying the system size is often impractical, scaling exponents are regularly obtained from the local roughness $w(l)$ (LW). The LW follows a power law, $w(l) \sim l^{\alpha \mathrm{loc}}$, where α_{loc} is known as the local roughness exponent. Systems in which the local and global roughness exponents are equal, $\alpha_{loc} = \alpha$, *i.e.* having no characteristic length scale besides the system size, follow the well-known Family–Vicsek (FV) ansatz [1]. In other systems however, the interface fluctuations follow different power laws at small and large scale. These systems are said to exhibit anomalous scaling (AS) since local and global roughness exponents are different ($\alpha_{loc} \neq \alpha$) [5]. AS has been observed in many experimental studies such as cracks in stone [6] and wood [7], CdTe films grown by hot wall epitaxy [8], glancing angle deposition films growth [9], and electrodeposition [10–12]. Interestingly, the growing mechanisms involved in the last two experimental processes are non-local in nature and tend to favor the deposition at protuberances over valleys or grooves.

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Motivated by these findings, in this paper we present a simple 1D Monte Carlo growth model, with a site dependent attachment probability P that depends on the surface curvature, followed by surface diffusion. Mimicking the above mentioned trend, the surface curvature acts in this model as a destabilizing mechanism that favors the deposition at protuberances (negative curvature sites) over the one at valleys (positive curvature sites). As a result of the proposed destabilizing mechanism, the surface roughness can grow indefinitely with time (α is not defined). Simultaneously, the model presents an anomalous behavior that depends on the difference in particle incorporation between protuberances and valleys. We will focus our analysis on the dynamic scaling properties of the proposed growth model.

2. Dynamic scaling

Kinetic roughening is widely resorted to class of models in non-equilibrium interfacial growth. As generally observed, many interfaces roughen as a result of fluctuations that arise from a variety of sources, such as non-homogeneous particle flux and aggregation. The interface roughness grows with time, until eventually a steady state is attained in which it reaches a size-dependent saturation value.

Roughening is usually described in terms of the surface width $w(l, t)$, defined as the root-mean square fluctuations of the interface with respect to its average value,

$$w(l,t) = \left\langle \frac{1}{l}\sum_{1}^{l}\left(h(i,t) - \overline{h(i,t)}\right)^2 \right\rangle^{1/2}, \tag{1}$$

where $h(i, t)$ is the surface height measured from the flat substrate at position i and at time t, $\overline{h(i,t)}$ is the mean height of the interface at the same time, and l is the length over which w is measured.

In many cases, $w(l, t)$ takes a scale invariant form in space and time that is fully characterized by the FV scaling ansatz [13,14]

$$w(l,t) \sim l^{\alpha} \quad \text{for } l \ll l_c \tag{2a}$$

$$w(l,t) \sim t^{\beta} \quad \text{for } l \gg l_c. \tag{2b}$$

The fact that kinetic roughening is a dynamical critical process can be seen by defining the dynamical exponent $z = \alpha/\beta$. Thus, for $l_c \ll t^{1/z}$, w scales as l^{α} while, for $l \gg t^{1/z}$, it scales as t^{β}. The crossover between the two behaviors takes place at the lateral correlation length $l_c \sim t^{1/z}$. α, known as the roughness exponent, characterizes the surface morphology at steady state. This happens for times larger than $t_c \gg L^z$ when the lateral correlation length l_c has reached a value larger than the system size. β is called growth exponent and characterizes the short time behavior of the surface growth. The evolution of the resulting roughness can be fully described in this scenario with a couple of scaling exponents, which can be used to characterize quantitatively the spatial and temporal evolution of surfaces and identify the growth process. FV scaling is supposed to arise in systems in which the growth rules lead to self-affine interfaces (scale invariance), *i.e.*, there is no characteristic length besides the system size. FV scaling is illustrated in Fig. 1(a). For a given deposition time, $\log w$ is proportional to $\log l$ up to a value of l about l_c, and then saturates. The value of l_c increases with time and the saturation value of the roughness scales as t^{β}.

A useful tool to extract scaling exponents is the height–height correlation function HHCF, $C(l, t)$, which is defined as

$$C(l,t) = \left\langle (h(i+l,t) - h(i,t))^2 \right\rangle^{1/2}, \tag{3}$$

where $h(i + l, t)$ is the surface height at a distance l from a reference site i and the averaging is done over different reference sites. For a self-affine surface, $C(l, t)$ scales with l as

$$C(l,t) \approx l^{\alpha} f(t/l^{z}) \tag{4}$$

where $f \sim$ constant for $L \gg l$.

Another useful tool is the power spectrum (PS) or surface structure factor $S(k, t)$ defined as

$$S(k,t) = \langle H(k,t)H(-k,t)\rangle, \tag{5}$$

being $H(k, t)$ the kth Fourier mode of the surface height deviation around its spatial average for a given time t

$$H(k,t) = \frac{1}{L^{1/2}}\sum_{j}\left[h(j,t) - \overline{h}(t)\right]\exp(ikx). \tag{6}$$

The dynamic scaling hypothesis implies that

$$S(k,t) = k^{-(2\alpha+1)} g(t/k^{-z}) \tag{7}$$

with $g(u) = t(2\alpha + 1)/z$ for $u \ll 1$, and $g(u) = constant$ for $u \gg 1$. Then:

$$\begin{aligned} S(k,t) &\sim k^{-(2\alpha+1)} \quad u \ll 1 \\ S(k,t) &\sim t^{(2\alpha+1)/z} \quad u \gg 1 \end{aligned} \tag{8}$$