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Bayesian estimation and model selection for spatial Durbin error model with finite distributed lags☆

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ABSTRACT

In this paper we investigate a spatial Durbin error model with finite distributed lags and consider the Bayesian MCMC estimation of the model with a smoothness prior. We study also the corresponding Bayesian model selection procedure for the spatial Durbin error model, the spatial autoregressive model and the matrix exponential spatial specification model. We derive expressions of the marginal likelihood of the three models, which greatly simplify the model selection procedure. Simulation results suggest that the Bayesian estimates of high order spatial distributed lag coefficients are more precise than the maximum likelihood estimates. When the data is generated with a general declining pattern or a unimodal pattern for lag coefficients, the spatial Durbin error model can better capture the pattern than the SAR and the MESS models in most cases. We apply the procedure to study the effect of right to work (RTW) laws on manufacturing employment.

1. Introduction

Spatial econometric models applied in regional science and geography have been receiving more attention in various areas of economics. The most popular spatial econometric model is the spatial autoregressive (SAR) model. The SAR model implies a geometrical decay pattern of spatial spillover effects or externalities from levels of neighbors' exogenous characteristics in its reduced form. A geometrical decay pattern of spatial distributed lags is not the only choice of modeling spatial externalities. There are other models which display a different pattern of spatial externalities. Recently, LeSage and Pace (2007) introduce the matrix exponential spatial specification (MESS) model, which exhibits an exponential declining pattern of spatial externalities. These two spatial lag patterns do not exhaust other possible patterns. Furthermore, both the SAR model and the MESS model incorporate global spatial externalities in the sense that they relate all the neighbors in the system to each other. If one wants to capture local spatial externalities,[1] these two models might not be appropriate. In practice, if one wants to incorporate local externalities in the model and there were no formal theoretical guidance on which pattern of spatial externalities to choose, a possible solution is to propose a spatial Durbin error model (SDEM)[2] with finite distributed lags of exogenous regressors, which does not impose strong restrictions on the pattern of spillover effects or externalities. Then we are facing a non-nested model selection problem among a SDEM model, the SAR model and the MESS model. Hence, in addition to the estimation of a SDEM model, it is of interest to construct a model discrimination procedure for them.

In this paper we propose a finite lag SDEM model with a smoothness prior in order to accommodate more flexible patterns of local spatial externalities. We consider the Bayesian MCMC estimation of the SDEM model and the corresponding Bayesian model selection

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[1] As mentioned by Anselin (2003), local spatial externalities would be appropriate when the proper spatial range of the explanatory variables is the location itself and its immediate neighbors. For more discussion regarding global and local spatial externalities, see Anselin (2003).

[2] We thank two referees for pointing out that the original name "SLX" for the model is somehow confusing. We have changed the name of the model to SDEM throughout the paper. Related literature about the SDEM model are mentioned in Section 2.

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procedure for the SDEM model, the SAR model and the MESS model. We focus on Bayesian estimation because a direct maximum likelihood (ML) estimation for high order lag coefficients of the SDEM model might be imprecise due to multicollinearity among lagged regressors.[3] As motivated by Shiller's smoothness prior for distributed lag model, we may impose a smoothness prior on the lag coefficients in order to obtain better estimates.

For model selection among non-nested models, both classical approach and Bayesian approach are available in the literature. However, the classical non-nested tests might be unreliable due to imprecise estimates for high order lag coefficients of a SDEM model. For Bayesian approach, Zellner (1971) has set forth the basic theory and the model selection procedure, which involves calculating and comparing the posterior probabilities of competitive models and is feasible for competitive non-nested models. With posterior probabilities for competitive models one can see whether those models are close competitors or one model just dominates all others. Hepple (1995a,b) extends the Bayesian model selection procedure into non-nested spatial models.[4] In particular, he has derived expressions of marginal likelihoods for a number of spatial models including the SAR model and the spatial error model, which greatly simplify the calculation of model posterior probabilities. LeSage and Pace (2007) derive an expression of the marginal likelihood of the MESS model, which could be used to produce Bayesian model comparison procedure for the MESS model and other models. LeSage and Parent (2007) also extend the Bayesian model selection procedure for linear regression models into the SAR model and the spatial error model. Their focus is on comparing models with different matrices of explanatory variables.[5] We can also derive an expression of the marginal likelihood of the SDEM model, which simplifies the model selection procedure.

The paper is organized as follows: Section 2 introduces the SDEM model with a smoothness prior and specifies the model selection problem. Section 3 considers Bayesian MCMC estimation of the SDEM model. Section 4 discusses Bayesian model selection procedure. Section 5 summarizes simulation results to investigate sampling properties of our Bayesian estimation method and model selection procedure. Section 5 includes an empirical study of the effect of right to work (RTW) on manufacturing employment. Conclusions are drawn in Section 6. Technical details and tables are given in Appendix A.

2. The models

Consider a spatial autoregressive (SAR) model

$$Y_n = \lambda W_n Y_n + c l_n + X_n \beta + V_n \tag{2.1}$$

where l_n is a $n \times 1$ column vector of ones, and c is the coefficient of the intercept term. X_n is a $n \times k$ dimensional matrix of nonstochastic exogenous variables. W_n is a spatial weight matrix with known constants with a zero diagonal. The error terms in $V_n = (v_{n1}, v_{n2}, \ldots, v_{nn})'$ are assumed to be i.i.d normally distributed with mean 0 and variance σ^2. The reduced form of the SAR model reviews its implication in spatial externalities in terms of a geometric declining pattern of spatial distributed lags on exogenous regressors,

$$Y_n = (I_n - \lambda W_n)^{-1} c l_n + X_n \beta + \sum_{q=1}^{\infty} W_n^q X_n \lambda^q \beta + (I_n - \lambda W_n)^{-1} V_n. \tag{2.2}$$

Here the nonzero elements of rows of W_n with $q \geq 1$ represent the qth order neighbors. Then the specification in Eq. (2.2) has spillover effects or externalities generated by the regressors xs from one's different level of neighbors being geometrically declining.

As an alternative to the SAR specification, LeSage and Pace (2007) introduce the MESS model with the specification $S_n^{ex}(\mu) Y_n = c l_n + X_n \beta + V_n$, of which the reduced form is

$$Y_n = S_n^{ex}(\mu)^{-1} c l_n + S_n^{ex}(\mu)^{-1} X_n \beta + S_n^{ex}(\mu)^{-1} V_n, \tag{2.3}$$

where $S_n^{ex}(\mu) = e^{\mu W_n} = I_n + \sum_{q=1}^{\infty} \frac{1}{q!} (\mu W_n)^q$. The model introduces an exponential decay pattern of spatial externalities via its spatial distributed lags on regressors.

However, according to Eqs. (2.2) and (2.3), both the SAR model and the MESS model may be rather "restrictive" because they impose strong restrictions on the pattern of spatial externalities. Moreover, Eqs. (2.2) and (2.3) are infinite summations for lagged regressors $W_n X_n$s, implying that both models only incorporate global spatial externalities because they relate all the neighbors in the system to each other. To allow for a more flexible pattern of spatial externalities and incorporate local spatial externalities, a possible specification is the following finite spatial distributed lag model:

$$\begin{aligned} Y_n = {} & c l_n + X_n \beta + W_n X_n \beta_1 + W_n^2 X_n \beta_2 + \ldots + W_n^m X_n \beta_m \\ & + (I_n - \rho W_n)^{-1} V_n. \end{aligned} \tag{2.4}$$

Here we combine local externalities for X_n with global externalities for V_n in the model.[6] We do not impose strong restrictions on the lag coefficients $\beta_1, \ldots, \beta_m$. As a result, the model should be able to accommodate more flexible patterns of spatial externalities. LeSage and Pace (2009) label the model in Eq. (2.4) as a spatial Durbin error model (SDEM). Some literatures have discussed other versions of the SDEM model, for example, Lacombe et al. (2012) and LeSage and Christina (2012). However, neither of them includes higher order spatial distributed lag terms of X_n in the model.[7] In empirical research, we usually do not have an economic theory that tells us which pattern of spatial externalities to choose. We are facing a non-nested model selection problem among the SDEM model, the SAR model and the MESS model.

However, one concern about the SDEM model in Eq. (2.4) is that, when we use the ML method to directly estimate the model, estimates of high order coefficients βs might be poor due to possible multicollinearity among the lagged regressors $X_n, W_n, X_n, \ldots, W_n^m X_n$.[8] In practice, one might expect that those coefficients would change smoothly, so we borrow the idea of smoothness prior from Shiller (1973) to impose some random restrictions on the lag coefficients and use the Bayesian MCMC method to estimate the SDEM model. For model selection, we follow the procedure advocated in Zellner (1971) and compare the posterior probabilities for the three models.

[3] There might be multicollinearity among the lagged exogenous regressors in the SDEM model. This is similar to the source of multicollinearity in the spatial Durbin model (Anselin, 1988). See Section 2 for more discussions.

[4] Hepple has considered a bunch of non-nested model selection problems for spatial models. For example, comparing SAR models with different spatial weight matrices or comparing spatial error models with different spatial weight matrices. For more details, see Hepple (1995a,b).

[5] LeSage and Parent deal with the cases where the number of possible models that consist of different combinations of candidate explanatory variables is too large. Calculation of posterior probabilities for all models is difficult. They rely on a Markov Chain Monte Carlo model composition methodology proposed by Madigan and York (1995). For more details, see LeSage and Parent (2007).

[6] Here local externalities are incorporated by the lagged exogenous regressors $W_n X_n$s while global externalities are captured by $(I_n - \rho W_n)^{-1} V_n$. The model in Eq. (2.4) is also a generalized version of Eq. (30) in Anselin (2003), with higher order lagged exogenous regressors $W_n X_n$s. See Anselin (2003) for more discussions.

[7] Lacombe et al. (2012) only include the first order spatial distributed lag in their SDEM model while LeSage and Christina (2012) include two different W_ns in the model.

[8] The estimates of the lag coefficients might have a large variance.