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# Aggregation systems for sales forecasting☆

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ABSTRACT

Sales forecasting consists of calculating the expected sales of a specific product or company. An important issue when dealing with sales forecasting is the calculation of the average sales, usually using the arithmetic mean or the weighted average. This study introduces new methods for calculating the average sales. These methods are two modern aggregation operators: the ordered weighted average, and the unified aggregation operator. The main advantage of this approach is the possibility to deal with uncertain and complex environments in a more complete way. The study develops some key examples through multi-person and multi-criteria techniques. The study also presents a numerical example regarding the calculation of the average sales of a product in a set of countries.



## 1. Introduction

The average sale is a number that considers a set of numerical sales and provides a representative value by using an averaging technique like the arithmetic mean or the weighted average. The average sale provides averaging results which are very useful in sales forecasting (Dalrymple, 1987; Mentzer & Cox, 1984) in many contexts, including the calculation of the averages sales of a product, company, sector, region, or country (Engle, Granger, & Hallman, 1989; Harrison, 1967).

The literature mentions many aggregation operators that have similar purposes as the weighted average, but in other contexts. A very common operator is the ordered weighted average (OWA) (Yager, 1988; Yager, Kacprzyk, & Beliakov, 2011). OWA is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. OWA is very useful for under or overestimating the information according to the attitudinal character of the decision maker. In this context, OWA permits to construct interval numbers (Merigó, 2012a). Many authors extend and generalize the OWA (Emrouznejad & Marra, 2014; Yager & Kacprzyk, 1997; Yager et al., 2011). Yager and Filev (1999) develop the induced OWA (IOWA) operator, which Merigó and Gil-Lafuente (2009) further generalize. Other authors use different techniques for integrating the OWA operator and the weighted average in the same formulation, including the weighted OWA (Torra, 1997), the hybrid average (Xu & Da, 2003), the importance OWA (Yager, 1998), the immediate weights (Merigó & Gil-Lafuente, 2012; Yager, Engemann, & Filev, 1995), and the OWA weighted average (OWAWA) (Merigó, Engemann, & Palacios-Marqués, 2013). Recently Merigó et al., (2015) present more general aggregation operators that are more flexible and can adapt to a wide range of situations. The authors call these operators unified aggregation operators (UAO). The probabilistic weighted average (PWA) (Merigó, 2012b) is particularly interesting. PWA combines objective and subjective information in the same formulation.

The aim of this study is to analyze the use of modern aggregation operators in the calculation of the average sales to develop better forecasting techniques. To do so, this study presents the OWA sales (OWAS) as a method for calculating the average sales in uncertain environments where the decision maker wants to under or overestimate the information. The main advantage of the OWAS is that this operator can consider any scenario, from the minimum to the maximum sales. Thus, the decision maker and the experts can represent their attitude from the most optimistic to the most pessimistic position. These options give the analysis more flexibility in dealing with the information. The study considers multi-person and multi-criteria techniques. The inclusion of general aggregation operators like the UAO operator allows a complete overview of complex environments.

This study examines some examples of average sales calculation in key situations like the average sales of a product or a company, or the

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average sales of a country or a region. One example is the calculation of the average sales in a set of countries at a world level. These aggregation techniques are very useful for a better representation of the data, and take into account the particular conditions of the problem under study. The main reason for their usefulness is that these aggregation techniques can deal with different sources of information in the same formulation, thus providing a more general approach for the analysis of complex data.

The rest of the study is as follows. Section 2: Review of basic preliminaries. Section 3: Key aggregation techniques for improving the calculation of the average sales. Section 4: Key examples' analysis. Section 5: A numerical example. Section 6: Findings.

## 2. Aggregation systems

Aggregation systems have increasing popularity in the literature (Beliakov, Pradera, & Calvo, 2007; Grabisch, Marichal, Mesiar, & Pap, 2011). Aggregation systems serve multiple purposes, providing summarized and representative results of a set of data. The most common aggregation systems are the arithmetic mean and the weighted average. With the OWA operator, the researcher does not know the weights (market share or opinions), so he or she aggregates the information according to his or her attitude. Another increasingly popular aggregation operator is the OWA operator (Yager, 1988). Scholars define OWA differently.

**Definition 1.** An OWA operator of dimension $n$ is a mapping $OWA$: $R^n \rightarrow R$ that has an associated weighting vector $W$ of dimension $n$ with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

where $b_j$ is the $j^{\text{th}}$ smallest of the $a_i$.

One of OWA's key advantages is that OWA provides a parameterized family of aggregation operators between the minimum and the maximum. Thus, any scenario from the most optimistic to the most pessimistic one is possible. This option is an advantage because uncertainty means that the future is unpredictable. Therefore, people may have different beliefs about the future depending on their opinion and attitude. The OWA represents these variations mathematically. The use of the minimum and the maximum in the OWA operator yields the classical interval numbers. The advantage is that, apart from these extreme results, OWA allows to consider results closer to the center but with some small degrees of optimism/pessimism. In decision making under uncertainty, OWA integrates the classical methods into a single formulation, being each particular method a specific expression of the OWA operator (Yager, 1988). The pessimistic criteria is found when $w_n = 1$ and $w_j = 0$ for all $j \neq n$. The optimistic criteria if $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The Laplace criteria when $w_j = 1/n$ for all $j$; and the Hurwicz criteria if $w_1 = \alpha$, $w_n = (1 - \alpha)$, and $w_j = 0$ for all $j \neq 1, n$.

Scholars extend the OWA operator in a wide range of directions. The UAO operator is a general framework for integrating different aggregation operators into a general formulation according to the degree of importance of each concept. This study defines UAO as follows.

**Definition 2.** A unified aggregation operator of dimension $m$ is a mapping $UAO$: $R^m \times R^n \rightarrow R$ that has an associated weighting vector $C$ of dimension $m$, such that:

$$UAO(a_1, \ldots, a_n) = \sum_{h=1}^{m} \sum_{i=1}^{n} C_h w_i^h a_i, \tag{2}$$

where $C_h$ is the weight that each sub-aggregation has in the system, with $C_h \in [0, 1]$, and $\sum_{h=1}^{m} C_h = 1$; $w_i^h$ is the $i^{\text{th}}$ weight of the $h^{\text{th}}$ weighting vector $W$ with $w_i^h \in [0, 1]$ and $\sum_{i=1}^{n} w_i^h = 1$.

The UAO operator includes a wide range of aggregation operators including the weighted average, the OWA, and the probabilistic OWAWA operator (Merigó, 2012a, 2012b). The UAO is more flexible than other operators because UAO can represent different types of weights in the same problem. Some important properties are the following.

- If $C_i = 1$, the system only considers one aggregation.
- If $C_i = 0$, the system does not consider this sub-aggregation.

Note that researchers can extend the UAO operator by adding some other key concepts like distance measures (Zeng, Merigó, & Su, 2013), fuzzy systems (Zeng, Su, & Le, 2012), power aggregations (Wei, Zhao, Wang, & Lin, 2013), Choquet integrals (Belles, Merigó, Guillen, & Santolino, 2014; Wei, Lin, Zhao, & Wang, 2014) and generalized multiple aggregations (Zhou, Chen, & Liu, 2013).

## 3. New aggregation systems in the average sales

The average sale is an important method for sales forecasting. The literature reports many methods for dealing with sales forecasting (Dalrymple, 1978; Huarng & Yu, 2014). Usually, scholars study the average sale with the arithmetic mean, if all the variables under study are equally important, or with the weighted average, if the variables have different degrees of importance. However, other approaches are feasible (Linares-Mustarós, Merigó & Ferrer-Comalat, 2015). The use of the OWA operator in the average sales yields the ordered weighted average sales (OWAS). OWAS deals with uncertain environments where decision makers do not know the importance of the variables. Instead, the decision maker uses his or her attitudinal character to weight the information. Thus, decision makers can consider any level of sales from the minimum to the maximum one. This option is useful for under or overestimating the sales according to an attitudinal position that a decision maker wants to consider in the analysis. The study defines OWAS as follows for a set of specific sales $S = \{s_1, s_2, \ldots, s_n\}$:

$$OWAS = \sum_{j=1}^{n} w_j S_j, \tag{3}$$

where $S_j$ is the $j^{\text{th}}$ smallest of the sales $s_i$ and $w_j$ is the weight such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Note that descending and ascending orders in the OWA operator interrelate by using $w_j = w^*_{n-j+1}$ (Merigó & Gil-Lafuente, 2013). If $w_n = 1$ and $w_j = 0$ for all $j \neq n$, results yield the maximum sale and if $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$, the minimum sale. If $w_j = 1/n$ for all $j$, the OWAS becomes the classical average sale.

To deal with situations where several experts provide their opinions, researchers can reformulate OWAS to form the multi-person OWAS (MP-OWAS) as follows:

$$MP{-}OWAS = \sum_{x=1}^{y} \sum_{j=1}^{n} e_x w_{jx} S_j, \tag{4}$$

where $e_x$ is the $x^{\text{th}}$ weight of each expert such that $e_x \in [0, 1]$ and $\sum_{x=1}^{y} e_x = 1$.

Researchers may include additional criterion to the OWAS like the use of several enterprises and countries forming the multi-person multi-country multi-enterprise OWAS (MPMCME-OWAS). The study defines MPMCME-OWAS as follows:

$$MP{-}MC{-}ME{-}OWAS = \sum_{x=1}^{y} \sum_{r=1}^{q} \sum_{k=1}^{p} \sum_{j=1}^{n} e_x u_{rx} v_{krx} w_{jkrx} S_j, \tag{5}$$

where $v_k$ is the $k^{\text{th}}$ weight of each enterprise such that $v_k \in [0, 1]$ and $\sum_{k=1}^{p} v_k = 1$, and $u_r$ is the $r^{\text{th}}$ weight of each country such that $u_r \in [0, 1]$ and $\sum_{r=1}^{q} u_r = 1$.

Similarly, by including additional concepts in the analysis, researchers could increase complexity of structures.