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An investigation into the vehicle routing problem with time windows and link capacity constraints

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A B S T R A C T

In this work, we investigate a new, yet practical, variant of the vehicle routing problem called the *vehicle routing problem with time windows and link capacity constraints* (VRPTWLC). The problem considers new constraints imposed on road links with regard to vehicle passing tonnage, which is motivated by a business project with a Hong Kong transportation company that transports hazardous materials (hazmats) across the city and between Hong Kong and mainland China. In order to solve this computationally challenging problem, we develop a tabu search heuristic with an adaptive penalty mechanism (TSAP) to help manage the company's vehicle fleet. A new data set and its generation scheme are also presented to help validate our solutions. Extensive computational experiments are conducted, showing the effectiveness of the proposed solution approach.

1. Introduction

One of the key decision support problems in supply chain management is the assignment of vehicles to specific customers and the generation of a supply schedule under a set of constraints. The vehicle routing problem with time windows (VRPTW) is a well-known operational research problem in transportation studies (see [16,23,24]). It involves a fleet of homogeneous vehicles, originating and terminating at a central depot, with capacity and travel time constraints, and servicing a set of customers with known demands and service-time windows. An important assumption in the traditional vehicle routing problems (VRPs) is that except for the distance, all road segments exhibit a homogeneous characteristic that has no impact on vehicle scheduling. However, this assumption is not always reasonable in real business applications. One example we encountered in a consultancy project for a transportation company in Hong Kong involves the transportation of hazardous materials (hazmats). Every year, thousands of tons of hazmats are transported across Hong Kong and between the city and mainland China. The key concern is safety since an accident may have significant impacts to human life and the natural environment, especially in large urban areas [28]. As a result, the local authority that administers road traffic ordinances and legislation specifies that safety levels for road links are different with respect to various environment conditions. The reason is that, for instance, if vehicles passing through a highly populated city area have even a small leakage, it may lead to disastrous consequences. Hence, transportation of hazmats is strictly restricted to certain levels in these areas. On the other hand, in areas with low population density, such as rural areas, vehicles are allowed to carry relatively larger quantities of such dangerous goods. Understandably, road links in this problem setting would have distinctly different safety requirements in terms of vehicle load (passing tonnage), and these requirements would in turn have a significant impact on routing schedules and transportation planning.

It is also worth mentioning that similar link-specific load requirements can sometimes be found in the transportation industry where truck overloading is recognized to be a major safety as well as cost concern. A freight vehicle, when overloading a link, can damage transport infrastructures [15,21,26]. The damage usually grows exponentially as the total load increases beyond a safety level, which usually leads to higher maintenance and repair costs and shortens the lifespan of transport links such as roads and bridges. In this work, we recognize and take this important but often overlooked requirement into consideration, and propose a new widely applicable variant of VRPTW, called the

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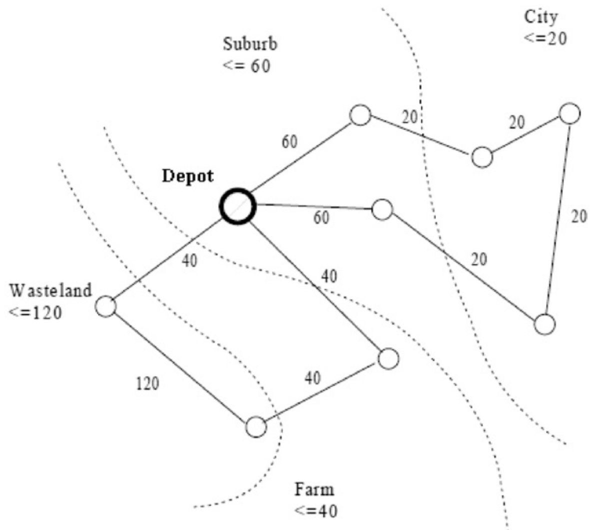

Fig. 1. A small-sized VRPTWLC instance.

vehicle routing problem with time windows and link capacity constraints (VRPTWLC). Fig. 1 shows an illustrative example of a small-sized VRPTWLC instance with eight customers and a depot. The underlying geography is divided into areas of four types: the city, the suburb, the farmland, and the wasteland. Boundaries are denoted by dashed lines. Values of load constraints imposed on road segments are listed, which are based on geographic characteristics of specific areas.

Vehicle routing problems with time windows have been widely studied as challenging combinatorial optimization problems during the past several decades. In addition to the landmark study by Solomon [23], recent advances have been made mainly on various heuristic solution approaches, as presented in extensive survey studies (see, for example, [3,4,8]). However, in the literature, only the recent work by Zografos and Androutsopoulos [27] has directly associated the VRPTW with transportation of dangerous goods. In the bi-objective model, a new risk term is proposed for each road segment, and work is then focused on an objective function to minimize the total travel time and the total risk. However, transportation risk on road links does not function as a "hard" requirement. Thus, as we observe, this bi-objective model can be effectively reduced to the traditional VRPTW by merging the risk with travel time to form a new cost term for each road segment. Other studies on transportation with safety requirements mainly involve risk modeling [20,5], the relationship between regulators and carriers [14] and road pricing [6], but none has looked into the effects of constraining vehicle load due to different requirements on various road links on an operational level. Therefore, this work provides a new perspective by introducing a practical set of "hard" link capacity constraints to the traditional VRPTW.

The remainder of this paper is organized as follows. We first define VRPTWLC and model it as a mixed-integer programming problem in Section 2. Then, a new one-stage tabu search with adaptive penalty mechanism (TSAP) is proposed in Section 3. Section 4 gives the computational results of our algorithms on newly generated benchmark data sets. The conclusion is presented in the last section.

2. Problem definition

The vehicle routing problem with time windows and link capacity is a new variant of VRPTW that considers vehicle scheduling between a depot and several customers. The fleet is considered to be homogeneous. The problem is defined on a network $G=(V,A)$ that consists of $n+2$ nodes. The customers are denoted by node $1,2,\dots,n$ ("the customer nodes"), and the single depot is denoted by both node 0 ("the starting depot") and node $n+1$ ("the returning depot"); the travel time between nodes 0 and $n+1$ is zero. We assume that there is exactly one arc between each ordered pair of nodes. Each arc represents a road segment and has a link capacity. The load of a passing vehicle is strictly restricted to the link capacity. The objective is to minimize the number of vehicles used and the total travel distance, provided that the link capacity is not exceeded on any of the road segments and the time window constraints are satisfied. Details of the constraints are given as follows.

1. Each vehicle has its own path, or route, which starts from node 0, visits one or more customer nodes, and terminates at node $n+1$. Because nodes 0 and $n+1$ denote the same depot, the terms "path" and "route" can be regarded as identical and interchangeable in this work. Zero travel time is defined between these two nodes.
2. Each customer node must be served once and only once to fulfill its positive demand. All customers' demands must be satisfied. The demand for nodes 0 and $n+1$ is zero.
3. Each customer node has a time window within which service must start. The vehicle has to wait if it arrives before the time window opens. Each customer node has a required service time. The vehicle cannot leave the node until the service is completed. Zero service time is defined for nodes 0 and $n+1$. Also known as the "scheduling horizon", a time window $[E,L]$ is associated with nodes 0 and $n+1$, where E and L represent the earliest departure time from the depot and the latest arrival time at the depot, respectively.
4. Link capacity cannot be exceeded by a passing vehicle.

The problem can then be formulated as a mixed-integer program. We assume all parameters are non-negative numbers, and the number of customers and number of vehicles are integers.

Parameters

$n \in \mathbb{Z}^+$ number of customers;
$K \in \mathbb{Z}^+$ number of vehicles;
$t_{ij} \in \mathbb{R}^+ \cup \{0\}$ travel time between nodes i and j $(0 \le i \le n, 1 \le j \le n+1, i \ne j)$;
$d_i \in \mathbb{R}^+ \cup \{0\}$ demand at node i $(0 \le i \le n+1)$;
$s_i \in \mathbb{R}^+ \cup \{0\}$ service time for node i $(0 \le i \le n+1)$;
$C \in \mathbb{R}^+$ capacity of each vehicle;
$a_i \in \mathbb{R}^+ \cup \{0\}$ lower bound of the customer's time window at node i $(0 \le i \le n+1)$;
$b_i \in \mathbb{R}^+ \cup \{0\}$ upper bound of the customer's time window at node i $(0 \le i \le n+1)$;
$r_{ij} \in \mathbb{R}^+$ link capacity between nodes i and j $(0 \le i \le n, 1 \le j \le n+1, i \ne j)$;
$F \in \mathbb{R}^+$ fixed cost per vehicle in use;
$M_{ij} \in \mathbb{R}^+$ a sufficiently large number $(0 \le i \le n, 1 \le j \le n+1, i \ne j)$.

Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ visits } i \text{ immediately before visiting node } j, \\ 0 & \text{otherwise.} \end{cases}$$

$w_{ik} \in \mathbb{R}$ the time when vehicle k starts to service node i.
$u_i \in \mathbb{R}$ the load of vehicle before servicing node i.