Electoral Studies 31 (2012) 852–860

Contents lists available at SciVerse ScienceDirect

Electoral Studies

journal homepage: www.elsevier.com/locate/electstud

Maximum likelihood estimation of structural VARFIMA models

Wen-Jen Tsay*,[1]

The Institute of Economics, Academia Sinica, 128 Academia Road, Sec. 2, Taipei, Taiwan, ROC

ARTICLE INFO

Article history:
Received 13 January 2012
Received in revised form 31 May 2012
Accepted 22 June 2012

Keywords:
Fractional cointegration
Structural VARFIMA
Durbin–Levinson algorithm

ABSTRACT

This paper considers the maximum likelihood estimation of a class of structural vector autoregressive fractionally integrated moving-average (VARFIMA) models. The structural VARFIMA model includes the fractional cointegration model as one of its special cases. We show that the conditional likelihood Durbin–Levinson (CLDL) algorithm of Tsay (2010a) is a fast and reliable approach to estimate the long-run effects as well as the short- and long-term dynamics of a structural VARFIMA process simultaneously. In particular, the computational cost of the CLDL algorithm is much lower than that proposed in Sowell (1989) and Dueker and Startz (1998). We apply the CLDL method to the Congressional approval data of Durr et al. (1997) and find that the long-run effect of economic expectations on Congressional approval is at least 0.5718, which is over twice the estimate of 0.24 found in Table 2 of Box-Steffensmeier and Tomlinson (2000). This paper also tests the divided party government hypothesis with the CLDL algorithm.

1. Introduction

This paper considers the maximum likelihood estimation of a class of structural vector autoregressive fractionally integrated moving-average VARFIMA (p,d,q), models. This model includes the fractional cointegration model as one of its special cases and has been investigated by Sowell (1989) and Dueker and Startz (1998). It also encompasses the stationary and invertible VARFIMA processes of Tsay (2010a) whereby all the data-generating processes (DGP) behind the structural VARFIMA models are stationary. The broad coverage of the structural VARFIMA model identifies itself as a useful workhorse for many political time series observations, including Box-Steffensmeier and Tomlinson (2000) and Clarke and Lebo (2003). In particular, in Section 4 of this paper we estimate the long-run effect of economic expectations on Congressional approval based on a 2-dimensional fractional cointegration model of Congressional approval data of Durr et al. (1997, DGW hereafter). The magnitude of the long-run effect should be the focus of the literature, because it signifies whether the economic prospects of the public strongly affect their support of the Congress. The use of the structural VARFIMA model allows us to simultaneously address the long-run effects as well as the short- and long-term dynamics characterized by the AR, MA, and the fractional differencing parameters.

This paper is strongly motivated by the observations in Box-Steffensmeier and Tomlinson (2000, p. 71) that the program of Dueker and Startz (1998) is extremely sensitive to the starting values and that their computation with Congressional approval data of DGW (1997) tends to get stuck in local minima. Another shortcoming of Sowell's (1989) algorithm is its heavy computational burden. Dueker and Startz (1998, p. 423) demonstrate that it takes 35 min on a 200-MHz PC for each iteration of the maximum likelihood estimation of a bivariate VARFIMA process with 121 observations and 18 parameters when implementing Sowell's (1989) algorithm.

* Tel.: +886 2 27822791x296; fax: +886 2 27853946.
E-mail address: wtsay@econ.sinica.edu.tw.

[1] The author thanks the Editor, Professor Clarke, and two anonymous referees for useful comments and suggestions. Part of the research was conducted when Tsay was a visiting scholar in the Department of Economics, University of Washington. The author would like to thank the financial and managerial support from the Fulbright Program. Research assistance from Shih-Tang Hwu is gratefully appreciated.

0261-3794/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.electstud.2012.06.007

This paper explains how the conditional likelihood Durbin–Levinson (CLDL) algorithm of Tsay (2010a) can be helpful to estimate the structural VARFIMA model efficiently. First, the CLDL algorithm is a one-step likelihood-based estimator and can estimate the aforementioned long-run effects as well as the short- and long-term dynamics of a structural VARFIMA process simultaneously. Because a one-step procedure is usually more efficient than a two-step or multiple-step procedure and is a consistent estimator if the model is correctly specified, the long-run effect estimate from the CLDL algorithm will generally be different from the one generated from Box-Steffensmeier and Tomlinson (2000) who employ a two-step procedure. Indeed, when applying the CLDL algorithm to test the divided party government hypothesis with the 80 observations of Congressional approval data of DGW (1997) based on a 3-dimensional VARFIMA (*p,d,q*) model in the following Section 5, we establish the first evidence that the disturbance term of the fractional cointegration model of Congressional approval might be a fractionally integrated process. This indicates that a flexible long memory process is required to capture the dynamic behavior of the data of Congressional approval and justifies the use of the structural VARFIMA model for this important issue of Political Science.

Second, the CLDL algorithm can evaluate the conditional likelihood function of the structural VARFIMA models exactly. This is in sharp contrast with the algorithm of Sowell (1989) and that of Dueker and Startz (1998), which are subject to a truncating error when the AR parameters are present.

The third advantage of the CLDL algorithm is that its computation is much faster than that proposed in Sowell (1989) and Dueker and Startz (1998), because it utilizes an efficient Durin–Levinson algorithm. Due to the high speed of computation, Tsay (2010a) conducts a Monte Carlo experiment to show the finite sample performance of the CLDL algorithm for 3-dimensional VARFIMA processes under a sample size of up to 400. Therefore, the use of the CLDL algorithm also resolves the comment of Lebo et al. (2000, p. 38) that “the only complaints about full maximum likelihood estimation concern its computationally intensive algorithm.”

The remaining parts of this paper are arranged as follows: Section 2 presents the structural VARFIMA (*p,d,q*) models. Section 3 explains the implementation of the CLDL algorithm for the structural VARFIMA process. We apply the CLDL methodology to the data of DGW (1997) in Section 4. The major task is to estimate the long-run effect of economic expectations on Congressional approval using various 2-dimensional VARFIMA models. Section 5 tests the divided party government hypothesis with a 3-dimensional structural VARFIMA model. Section 6 provides a conclusion.

2. Structural VARFIMA models

Consider the structural multivariate time series model with fractionally integrated errors:

$$\begin{cases} y_t = \alpha_1^\top D_t + \beta^\top X_t + u_t, \\ X_t = \alpha_2 D_t + \gamma^\top X_{t-1} + V_t, \end{cases} \quad or \quad \begin{bmatrix} y_t - \alpha_1^\top D_t - \beta^\top X_t \\ X_t - \alpha_2 D_t - \gamma^\top X_{t-1} \end{bmatrix} = \begin{bmatrix} u_t \\ V_t \end{bmatrix} = W_t, \tag{1}$$

where D_t is a vector of deterministic functions, including a constant or linear trend, α_1 is a vector of parameters, and α_2 is a matrix of parameters conformable to D_t. Here, u_t is a univariate fractionally integrated process of order d_1, and V_t is an $r-1$ dimensional time series with $r-1$ potentially different orders of fractional integratedness. This model has been considered by Baillie and Bollerslev (1994), Cheung and Lai (1993), and Dueker and Startz (1998). When $r=2$ and $\gamma = 1$, we can easily see the well-known fractional cointegration model belongs to one of its special cases.

The model in eq. (1) is considered in Sowell (1989) and later employed by Dueker and Startz (1998) to describe the joint behavior of U.S. and Canadian bond rates. Essentially, the idea behind the algorithm of Sowell (1989) and Dueker and Startz (1998) for the model in eq. (1) is to follow the spirit of the conditional likelihood function in Box and Jenkins (1976, Chapter 7). In other words, conditional on the structural parameters $\{\alpha_1, \alpha_2, \beta, \gamma\} = \Psi$, we can write down the likelihood of W_t, provided the probability density function of W_t is known. The associated conditional likelihood is denoted as $L(W_t|\Psi)$. This notation signifies the likelihood function is computed conditional on the value of Ψ.

In the literature starting with Sowell (1992), W_t is usually assumed to be generated as:

$$\Phi(B) diag(\nabla^d) W_t = \Theta(B) Z_t, \tag{2}$$

where $t = 1, 2, \ldots, T$, W_t is an r-dimensional vector of observations of interest, and $\Phi(B)$ and $\Theta(B)$ are finite order matrix polynomials in B (usual lag operator), such that:

$$\Phi(B) = \Phi_0 - \Phi_1 B - \ldots - \Phi_p B^p, \quad \Theta(B) = \Theta_0 + \Theta_1 B + \ldots + \Theta_q B^q, \quad \Phi_0 = \Theta_0 = I_r, \tag{3}$$

I_r is an $r \times r$ identity matrix, and the diagonal matrix $\mathrm{diag}(\nabla^d)$ is defined as:

$$diag(\nabla^d) = \begin{bmatrix} \nabla^{d_1} & 0 & \ldots & 0 \\ 0 & \nabla^{d_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \nabla^{d_r} \end{bmatrix}, \tag{4}$$

where $\nabla = 1-B$, and $d_i \in (-1/2, 1/2)$, for all $i = 1, 2, \ldots, r$. Here, $Z_t = (z_{1,t}, \ldots, z_{r,t})^\top$ in eq. (2) is an r-dimensional independent and identically distributed (i.i.d.) white noise process with a nonsingular covariance matrix Σ. The VARFIMA format naturally combines the feature of a univariate ARFIMA model and that of a VARMA process, thus providing a flexible modeling framework for empirical applications. See Sowell (1989) about the structural VARFIMA model.

We theoretically need to evaluate the autocovariance function of W_t before we can compute the conditional log-likelihood function exactly. This task is not trivial for the VARFIMA process. As clearly pointed out in Tsay (2010a), the presence of the AR parameters greatly complicates the