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A simplified model for the steady-state biofilm-activated sludge reactor

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Abstract

A simplified mathematical model is proposed to describe the steady-state completely mixed biofilm-activated sludge reactor (hybrid reactor). The model is derived based on Monod kinetic expressions and the Fickian diffusion law in biofilm. In addition, it considers all the essential concepts that describe the two types of growth (suspended and attached) and the competition between them for limiting substrate. Also the present study has been extended to investigate simple and accurate mathematical expressions for describing the substrate diffusion in biofilm (*J*). The expression for substrate flux has an explicit solution, which may be useful in the proposed model and many other applications. The application of the model for the hybrid system has been explained for a given set of data and verified by comparison with another solution. Also the model was applied to experimental results for a trace level of suspended biomass concentration (*X*). It was found that the biofilm flux (*J*) is the key factor in the model prediction, hence the accuracy of the model output is influenced by the accuracy of *J*. Compared with other solutions for such systems the model is simple, easy to use, and provides an accurate tool for describing such systems based on fundamental principles.

Keywords: Activated sludge; Biofilm; Hybrid reactor; Mathematical model; Steady state; Wastewater

1. Introduction

Aerobic biological systems for wastewater treatment are based on either suspended or attached growth. Recently, the combination of the two types of growth in one system has been found to be advantageous for improvement of the efficiency and/or capacity of existing treatment plants. Incorporating biofilm within the activated sludge process (ASP) is one of the most commonly adopted configurations of the hybrid reactors. In this system the biofilm is grown on a fixed or movable carrier in the biological reactor. As a result of this combination, the substrate removal efficiency is significant even at low temperature (Lessel, 1994; Tsuno et al., 1992; Hamoda and AL-Sharekh, 2000). Further the system stability and the sludge properties are improved (Martinez and Luciano, 1992; Jianlong et al., 2000). Although the system has been incorporated in many areas, evaluation of its performance based on kinetic principles has not yet been elucidated. The system is more complex compared to the biofilm or the pure suspended growth reactor. Analysis of the system is difficult due to the need for biofilm analysis, differentiation between the suspended and the attached growth behavior, and the complexity of the combined system.

At present, hybrid reactors are designed based on a recommended ratio of the biofilm carrier to the reactor associated with a desired removal efficiency. This ratio is obtained from field experience or experimental results. Gebara (1999) proposed a design technique by considering the system as two separate reactors (biofilm and ASP) run in series. These methods of design neither give a precise result nor a good estimation of system parameters. Lee (1992) presented a mathematical model to describe this system, but he assumed that sludge was wasted only from the reactor with concentration (*X*) and not from the settling tank as a simplifying step.

The present study has focused mainly on developing a simple mathematical model that can be used to design and

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Nomenclature

a	specific surface area of biofilm reactor (L^{-1})
b_s	specific shear loss rates (T^{-1})
b_t	first order decay and shear loss rates, b_s+K_d (T^{-1})
D_f	molecular diffusion coefficient in biofilm ($L^2\,T^{-1}$)
D_w	molecular diffusion coefficient in water ($L^2\,T^{-1}$)
F/M	food to the biomass ratio (T^{-1})
J	substrate flux into biofilm ($M\,L^{-2}\,T^{-1}$)
J^*	dimensionless substrate flux, $J/\sqrt{(K_s K X_f D_f)}$
K	maximum specific rate of substrate utilization (T^{-1})
K_d	specific decay rate (T^{-1})
K_s	Monod half-velocity coefficient ($M\,L^{-3}$)
L	thickness of the stagnant layer (L)
L^*	dimensionless thickness of the stagnant layer, $L\sqrt{(KX_f)/(K_s D_f)}(D_f/D_w)$
Q	flow rate into the reactor ($L^3\,T^{-1}$)
Q_e	flow rate of the treated effluent ($L^3\,T^{-1}$)
Q_w	flow rate of the wasted sludge ($L^3\,T^{-1}$)
R	fraction of substrate utilized by suspended biomass
R^*	ratio of the active biofilm to the suspended biomass in the reactor
S	effluent substrate concentration ($M\,L^{-3}$)
S^*	dimensionless effluent substrate concentration, S/K_s
S_{min}	minimum substrate concentration that can sustain biofilm, $K_s/(YK/b_t-1)$
S^*_{min}	dimensionless minimum substrate concentration that can sustain biofilm, $1/(YK/b_t-1)$
S_o	influent substrate concentration ($M\,L^{-3}$)
S^*_o	dimensionless influent substrate concentration, S_o/K_s
V	liquid volume of the reactor (L^3)
X	suspended biomass concentration ($M\,L^{-3}$)
X_e	biomass concentration in the treated effluent (M/T)
X_f	biofilm microbial density ($M\,L^{-3}$)
X_u	biomass concentration of the wasted sludge ($M\,L^{-3}$)
Y	yield coefficient
θ	HRT, hydraulic retention time in the reactor (T)
θ_c	SRT, sludge residence time of the suspended biomass (T)

control such systems. The model was developed based on Monod kinetic expressions for both types of growth simultaneously and the Fickian diffusion law for biofilm combined with the basic equations of the hybrid reactor. Therefore it can be used for municipal as well as for industrial wastewater. In addition the study aimed at investigating a simple and explicit expression for substrate flux into the biofilm (J), which is the main parameter in the model. Using this model, the hybrid system can be designed and controlled accurately based on fundamental principals and the kinetic parameters for both the attached and suspended growth. Further, the model is easy to use and can be considered a good tool to explain the performance and the characteristics of such systems. Knowledge of hydraulic retention time (θ), influent substrate concentration (S_o), stagnant liquid layer thickness (L), minimum substrate concentration that can maintain the biofilm growth (S_{min}), the desired value of substrate concentration in the bulk phase (S) and kinetic constants permit computation of the suspended biomass concentration (X). After computation of X, other parameters such as food to biomass ratio (F/M) and wasted-sludge mass ($Q_w X_u + Q_e X_e$) can be obtained. Further the ratio of the active biofilm to the suspended biomass in the reactor R^* and the fraction of substrate used by both types of growth can be determined.

2. Development of the model

The process diagram of the aerobic hybrid system used for the present study is shown in Fig. 1(a), while the schematic of the system is shown in Fig. 1(b). The system is assumed to run under a steady-state condition for biomass and substrate with a rate limiting substrate concentration in the reactor. Kinetics for both suspended and attached growth are assumed to be the same (Williamson and McCarty, 1976; Lee, 1992; Gebara, 1999). For the entire system, the mass balance equations for biomass and substrate are as follows.

2.1. Substrate balance

The substrate balance for the hybrid system can be written as follows

$$\frac{dS}{dt} = Q(S_o - S) - aVJ - VX\frac{KS}{K_s + S} \qquad (1)$$

where dS/dt, the growth rate of substrate concentration in the bulk; Q, flow rate; a, specific surface area of the biofilm; V, reactor volume; K, maximum specific rate of substrate utilization; and K_s, Monod half-velocity coefficient.